Revision – Unit2 Functions

(Type of Functions, The Domain and Range of a function, Inverse Functions, Composite Functions, Transformations of functions)

The function f is defined by $f(x) = 3 + 6x - 2x^2$ for $x \in \mathbb{R}$.

Nov 2024 /11/ Q11

(a) Express $f(x)$ in the form $a - b(x - c)^2$, where a, b and c are constants, and state the range of f. **[3]**

(b) The graph of $y = f(x)$ is transformed to the graph of $y = h(x)$ by a reflection in one of the axes followed by a translation. It is given that the graph of $y = h(x)$ has a minimum point at the origin.

Give details of the reflection and translation involved. **[2]**

The function g is defined by $g(x) = 3 + 6x - 2x^2$ for $x \leqslant 0$.

(c) Sketch the graph of $y = g(x)$ and explain why g is a one-one function. You are **not** required to find the coordinates of any intersections with the axes. [2]

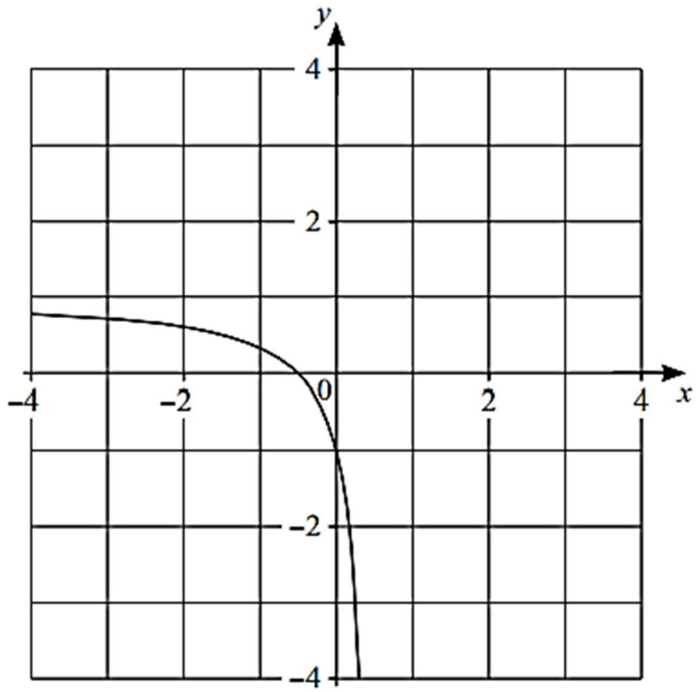
(d) Sketch the graph of $y = g^{-1}(x)$ on your diagram in **(c)**, and find an expression for $g^{-1}(x)$. You should label the two graphs in your diagram appropriately and show any relevant mirror line. [4]

The function f is defined by $f(x) = \dfrac{2x+1}{2x-1}$ for $x < \frac{1}{2}$.

Nov 2024 /12/Q5

(a) **(i)** State the value of $f(-1)$. [1]

(ii)



The diagram shows the graph of $y = \mathrm{f}(x)$. Sketch the graph of $y = \mathrm{f}^{-1}(x)$ on this diagram. Show any relevant mirror line. [2]

(iii) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of the function f^{-1}. [4]

The function g is defined by $g(x) = 3x + 2$ for $x \in \mathbb{R}$.

(b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$. [3]

The function $f(x) = 3x^2 - 12x + 14$ is defined for $x \geqslant k$, where k is a constant.

(b) Find the least value of k for which the function f^{-1} exists. [1]

Nov 2024 /13/Q8

For the rest of this question, you should assume that k has the value found in part **(b)**.

(c) Find an expression for $f^{-1}(x)$. [3]

(d) Hence or otherwise solve the equation $ff(x) = 29$. [3]

The functions f and g are defined for all real values of x by Mar2024/12/Q9

$$f(x) = (3x-2)^2 + k \quad \text{and} \quad g(x) = 5x - 1,$$

where k is a constant.

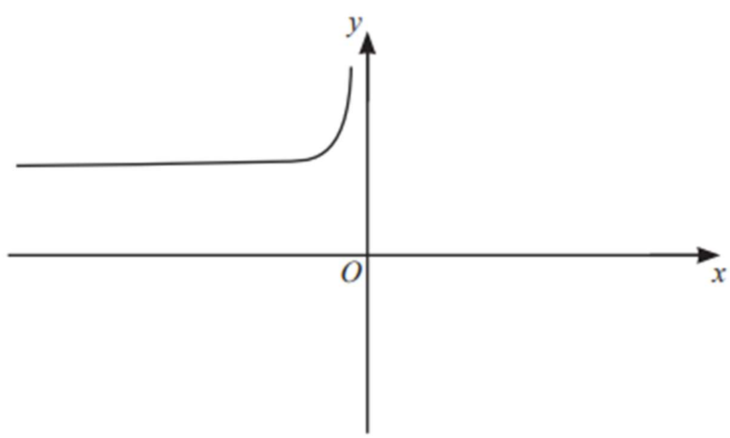
(a) Given that the range of the function gf is $gf(x) \geqslant 39$, find the value of k. [4]

(b) For this value of k, determine the range of the function fg. [2]

(c) The function h is defined for all real values of x and is such that $gh(x) = 35x + 19$.

Find an expression for $g^{-1}(x)$ and hence, or otherwise, find an expression for $h(x)$. [3]

May2024/11/Q6



The function f is defined by $f(x) = \frac{2}{x^2} + 4$ for $x < 0$. The diagram shows the graph of $y = f(x)$.

(a) On this diagram, sketch the graph of $y = f^{-1}(x)$. Show any relevant mirror line. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

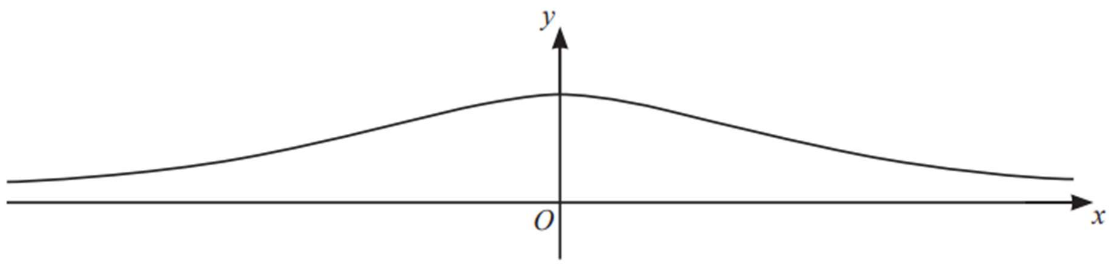
(c) Solve the equation $\mathrm{f}(x) = 4.5$. [1]

(d) Explain why the equation $\mathrm{f}^{-1}(x) = \mathrm{f}(x)$ has no solution. [1]

The function f is defined as follows: May2024/12/Q4

$$\mathrm{f}(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

(a) Find an expression for $\mathrm{f}^{-1}(x)$. [1]



The diagram shows the graph of $y = g(x)$ where $g(x) = \dfrac{1}{x^2 + 2}$ for $x \in \mathbb{R}$.

(b) State the range of g and explain whether g^{-1} exists. [2]

The function h is defined by $h(x) = \dfrac{1}{x^2 + 2}$ for $x \geqslant 0$.

(c) Solve the equation $hf(x) = f\left(\dfrac{25}{16}\right)$. Give your answer in the form $a + b\sqrt{c}$, where a, b and c are integers. [4]

The function f is defined by $f(x) = 10 + 6x - x^2$ for $x \in \mathbb{R}$. May2023/12/Q11

(a) By completing the square, find the range of f. [3]

The function g is defined by $g(x) = 4x + k$ for $x \in \mathbb{R}$ where k is a constant.

(b) It is given that the graph of $y = g^{-1}f(x)$ meets the graph of $y = g(x)$ at a single point P.

Determine the coordinates of P. [6]

A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$. Mar2024/13/Q2

Stretch parallel to the x-axis with scale factor $\frac{1}{2}$

Reflection in the y-axis

Stretch parallel to the y-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a, b and c are constants. [4]

The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$. Mar2023/12/Q9

(a) State the range of f. [1]

(b) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$. Mar2023/12/Q2

Stretch parallel to the x-axis with scale factor $\frac{1}{2}$

Reflection in the y-axis

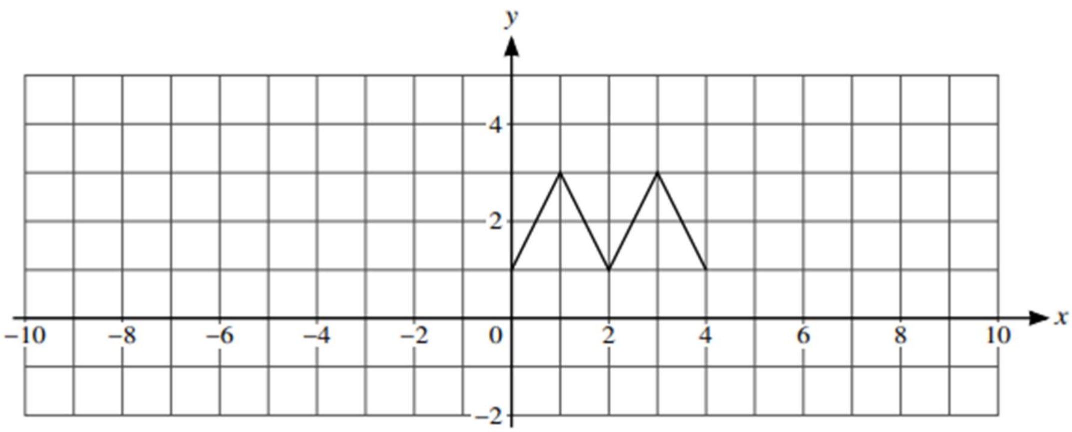
Stretch parallel to the y-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a, b and c are constants. [4]

The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x-direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Nov2022/12/Q5

(a) The diagram below shows the graph of $y = f(x)$.

On the diagram sketch the graph of $y = g(x)$. [3]



(b) Find an expression for $g(x)$ in terms of $f(x)$. [2]

(a) Express $f(x)$ in the form $-2(x+a)^2+b$, where a and b are integers. [2]

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(b) Find the range of f. [1]

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(c) Find an expression for $f^{-1}(x)$. [3]

Functions f and g are defined as follows:

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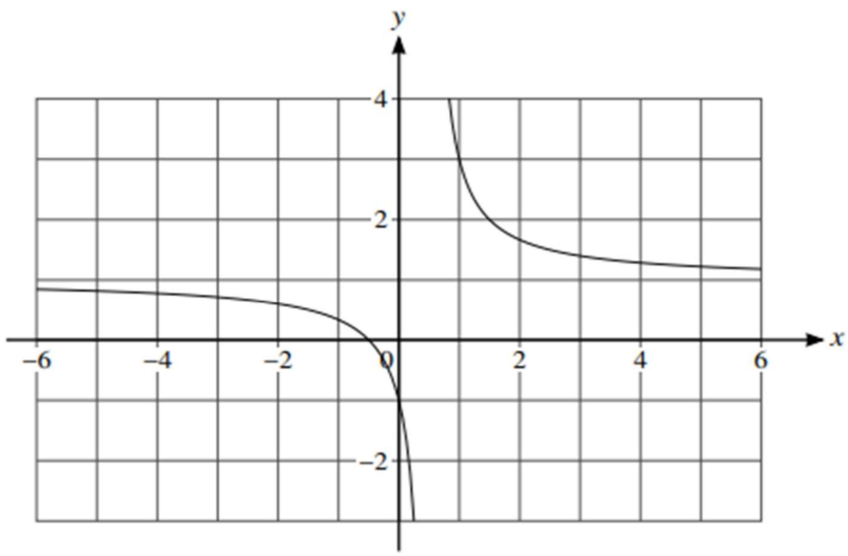
$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \tfrac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1}. [1]

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(b) Find an expression for $f^{-1}(x)$. [3]

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(c) Find $gf^{-1}(3)$. [2]

(d) Explain why $g^{-1}(x)$ cannot be found. [1]

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(e) Show that $1 + \dfrac{2}{2x - 1}$ can be expressed as $\dfrac{2x + 1}{2x - 1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x- and y-axes. [6]

The functions f and g are defined by

$$f(x) = x^2 \quad \text{for } x \in \mathbb{R},$$
$$g(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

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(b) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, making clear the order in which the transformations are applied. [4]

The one-one function f is defined by $f : x \mapsto -3x^2 + 12x + 2$ for $x \leqslant k$.

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(b) State the largest possible value of the constant k. [1]

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It is now given that $k = -1$.

(c) State the range of f. [1]

(d) Find an expression for $f^{-1}(x)$. [3]

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The result of translating the graph of $y = f(x)$ by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is the graph of $y = g(x)$.

(e) Express $g(x)$ in the form $px^2 + qx + r$, where p, q and r are constants. [3]

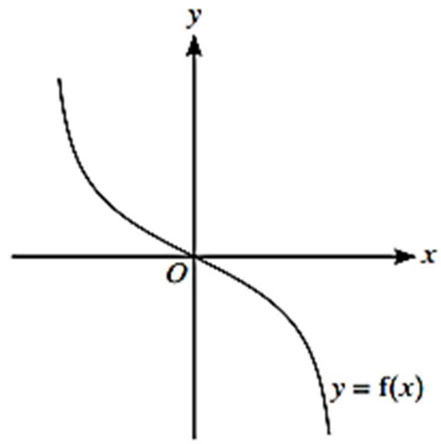
The function f is defined as follows: Nov2021/12/Q3

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of $ff(5)$. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

Nov2021/13/Q6



The diagram shows the graph of $y = f(x)$.

(a) On this diagram sketch the graph of $y = f^{-1}(x)$. **[1]**

It is now given that $f(x) = -\dfrac{x}{\sqrt{4 - x^2}}$ where $-2 < x < 2$.

(b) Find an expression for $f^{-1}(x)$. **[4]**

The function g is defined by $g(x) = 2x$ for $-a < x < a$, where a is a constant.

(c) State the maximum possible value of a for which fg can be formed. [1]

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(d) Assuming that fg can be formed, find and simplify an expression for $fg(x)$. [2]

Functions f and g are defined as follows:

Nov2021/13/Q6

$$f(x) = (x-2)^2 - 4 \text{ for } x \geqslant 2,$$
$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f. [1]

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(b) Find $f^{-1}(x)$. [2]