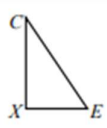


|  |            |   |                                     |
|--|------------|---|-------------------------------------|
| Use correct sector area formula  | <b>M1</b>  |   | Nov/9<br>709/11<br>/2024/<br>Q3     |
| Obtain $\frac{1}{2} \times 15^2 \times \frac{2}{5} \pi - \frac{1}{2} \times x^2 \times \frac{2}{5} \pi = \frac{209}{5} \pi$ or equivalent                        | <b>A1</b>  |   |                                     |
| Obtain $[x] = 4$   | <b>A1</b>  | AWRT 4.00.  |                                     |
| Use correct arc length formula twice   | <b>M1</b>  |   |                                     |
| Obtain $22 + \frac{38}{5} \pi$   | <b>A1</b>  | OE. Must be in terms of $\pi$ .<br>Like terms must be collected.<br>Not from a rounded value of $x$ .                 |                                     |
|  | <b>5</b>   |   |                                     |
| [Perimeter =] $r + r\theta + r + 2r \times 2\theta + r + r\theta + r$ $[= 4r + 6r\theta]$  | <b>B1</b>  |   | Nov/<br>9709/1<br>2/<br>2024/<br>Q6 |
| [Area =] $\frac{1}{2} r^2 \theta + \frac{1}{2} (2r)^2 \times 2\theta + \frac{1}{2} r^2 \theta$ $[= 5r^2 \theta]$   | <b>B1</b>  |   |                                     |
| $4r + 6r\theta = 14$ and $5r^2 \theta = 10$  | <b>M1*</b> | $ar + b\theta = 14$ and $cr^2 \theta = 10$ where $a, b$ and $c$ are constants $\neq 0$ .<br>Terms may be uncollected. |                                     |
| <b>EITHER</b>  |            |   |                                     |
| $5r^2 \frac{14 - 4r}{6r} = 10$ or $4r + 6r \left( \frac{10}{5r^2} \right) = 14$  | <b>DM1</b> | Eliminate $\theta$ to get an equation in $r$ .  |                                     |
| $[\Rightarrow 2r^2 - 7r + 6 = 0 \Rightarrow] (r - 2)(2r - 3) = 0$  | <b>DM1</b> | Factorise or other accepted method for solving their 3-term quadratic.  |                                     |
| <b>OR</b>  |            |   |                                     |
| $5 \left( \frac{14}{4 + 6\theta} \right)^2 \theta = 10$ or $4 \left( \sqrt{\frac{10}{5\theta}} \right) + 6 \left( \sqrt{\frac{10}{5\theta}} \right) \theta = 14$ | <b>DM1</b> | Eliminate $r$ to get an equation in $\theta$ .  |                                     |
| $[\Rightarrow 18\theta^2 - 25\theta + 8 = 0 \Rightarrow] (9\theta - 8)(2\theta - 1) = 0$   | <b>DM1</b> | Factorise or other accepted method for solving their 3-term quadratic.  |                                     |
| <b>Then</b>  |            |   |                                     |
| $r = 2$ and $\theta = 0.5$   | <b>B1</b>  | Condone extra answers $r = \frac{3}{2}$ and $\theta = \frac{8}{9}$ .  |                                     |
|  | <b>6</b>   |   |                                     |

|            |   |            |   |                                      |
|------------|---|------------|---|--------------------------------------|
| (a)        | Area of sector $BOF = \frac{1}{2} \times 20^2 \times (2\pi - 2.4) [= 776.63\dots]$  | <b>M1</b>  | Or combination of large semi-circle and small sector:<br>$\frac{1}{2} \times 20^2 \times \pi + \frac{1}{2} \times 20^2 \times (\pi - 2.4).$ | Nov/<br>9709/1<br>3/<br>2024/<br>Q7  |
|            | Length $BD = DF = 2 \times 20 \sin 0.6$ or $\sqrt{20^2 + 20^2 - 2 \times 20 \times 20 \cos 1.2}$<br>[= 22.58...]                                | <b>M1*</b> | Length of radius of small circles is acceptable for M1.   |                                      |
|            | Area of two semicircles = $\pi \times (20 \sin 0.6)^2 [= 400.64\dots]$  | <b>DM1</b> |   |                                      |
|            | Area of triangles = $2 \times \frac{1}{2} \times 20 \times 20 \sin 1.2 [= 372.81\dots]$   | <b>M1</b>  |   |                                      |
|            | Total area = 1550 [cm <sup>2</sup> ]  | <b>A1</b>  | Expect 1550.09 but accept AWRT to 3sf.  |                                      |
|            |   | <b>5</b>   |   |                                      |
| (b)        | $\frac{1}{2} \pi r^2 = 50\pi \Rightarrow r = 10$  | <b>B1</b>  | May be seen as $20 \sin \frac{\theta}{2}$ , where $\theta = \frac{\pi}{3}$ .  |                                      |
|            | $\Rightarrow \theta = \frac{\pi}{3}$  | <b>M1*</b> | OE<br>Finding $\theta$ using <i>their</i> $r$ . Allow working in degrees.   |                                      |
|            | Arc length of sector $BOF = 20 \times \left( 2\pi - \text{their} \frac{2\pi}{3} \right)$  | <b>DM1</b> |   |                                      |
|            | Total perimeter = $20 \times \left( 2\pi - \text{their} \frac{2\pi}{3} \right) + 2\pi \times \text{their} 10$                                   | <b>DM1</b> | Dependent on the first dM1.   |                                      |
|            | $\frac{140\pi}{3}$ or $46\frac{2}{3}\pi$  | <b>A1</b>  | Must be a single exact term.  |                                      |
|            | <b>5</b>  |            |   |                                      |
| (a)        | Angle $\theta = \frac{\pi}{2} - \cos^{-1} \frac{10}{15}$ or $\sin^{-1} \frac{10}{15} = 0.7297$  | <b>B1</b>  | Condone working in degrees if converted to radians at the end.<br>AG  | June/<br>9709/1<br>1/<br>2024/<br>Q7 |
|            |   | <b>1</b>   |   |                                      |
|            | (b) $BC = \sqrt{15^2 - 10^2} [= 11.18\dots \text{ or } 5\sqrt{5}]$  | <b>B1</b>  |   |                                      |
|            | Arc $AB = 15 \times 0.7297 [= 10.9455]$   | <b>B1</b>  |   |                                      |
|            | Perimeter = <i>their</i> $BC$ + <i>their</i> arc $AB$ + $25 + 5\pi$   | <b>M1</b>  |   |                                      |
|            | Perimeter = 62.8  | <b>A1</b>  | AWRT  |                                      |
|            | Area sector $AOB = \frac{1}{2} \times 15^2 \times 0.7297 [= 82.09]$   | <b>B1</b>  |   |                                      |
|            | Area = $\frac{1}{2} \times 10 \times \text{their } BC$ + <i>their</i> sector $AOB$ + $\frac{\pi}{4} \times 10^2$                                | <b>M1</b>  |   |                                      |
| Area = 217 | <b>A1</b>   | AWRT       |   |                                      |
|            | <b>7</b>  |            |   |                                      |
| (a)(i)     | <br>$\angle C = \frac{\pi}{6}$<br>$\angle E = \frac{\pi}{3}$ |            |   | June/<br>9709/1<br>2/<br>2024/<br>Q8 |
|            | $\frac{XE}{0.4} = \sin \frac{\pi}{6}$ or $\frac{XE}{0.4} = \cos \frac{\pi}{3}$ [ $XE = 0.2$ ]   | <b>M1</b>  | A correct trig expression involving $XE$ .<br>Do not condone a mixture of degrees and radians.  |                                      |
|            | Length $EF = 2 + 2 \times 0.2 = 2.4$  | <b>A1</b>  | AG  |                                      |
|            | <b>2</b>  |            |   |                                      |

|         |  |            |  |                                      |
|---------|--|------------|--|--------------------------------------|
| (a)(ii) | $[CX =] 0.4 \cos \frac{\pi}{6}$ or $0.4 \sin \frac{\pi}{3}$ or $\sqrt{0.4^2 - 0.2^2}$                                      | <b>B1</b>  | OE, SOI<br>Expect $\frac{\sqrt{3}}{5}$ or 0.3464.  |                                      |
|         | $[\text{Sector} =] \frac{1}{2} \times (0.4)^2 \times \frac{\pi}{3}$  | <b>B1</b>  | SOI<br>Expect 0.0838 or $\frac{2\pi}{75}$ . Allow use of $\frac{60}{360} \pi (0.4)^2$ .  |                                      |
|         | Either Area of <i>their</i> (rectangle + two triangles + two sectors)<br>Or Area of <i>their</i> (trapezium + two sectors) | <b>M1</b>  | Either implied by a correct answer or areas clearly labelled.<br>Expect 0.6928 + 0.06928 + 0.1676 or<br>$\frac{2\sqrt{3}}{5} + \frac{\sqrt{3}}{25} + \frac{4\pi}{75}$ .<br>Or 0.7621 + 0.1676 or $\frac{11\sqrt{3}}{25} + \frac{4\pi}{75}$ . |                                      |
|         | 0.930  | <b>A1</b>  | AWRT<br>Condone $\frac{11\sqrt{3}}{25} + \frac{4\pi}{75}$ .  |                                      |
| (b)     | [Length $AD =$ ] $2 + 2r$  | <b>B1</b>  | Must be seen alone or part of a list and not part of a product.  |                                      |
|         | [Arc length =] $r \times \frac{\pi}{3}$  | <b>B1</b>  | May be implied by $r \times \frac{\pi}{3} \times 2$ .<br>Must be seen alone or part of a list.   |                                      |
|         | [ $EF =$ ] $2 + 2r \sin \frac{\pi}{6}$ or $2 + 2r \cos \frac{\pi}{3}$ or $2 + r$   | <b>B1</b>  | Must be seen alone or part of a list and not part of a product.  |                                      |
|         | $[4 + 3r + \frac{2\pi r}{3} = 6$ leading to ] 0.393  | <b>B1</b>  | AWRT<br>Condone $\frac{6}{2\pi + 9}$ .<br>NB: Using $EF = 2.4$ gives 0.391.  |                                      |
|         | <b>4</b>   |            |  |                                      |
| (a)     | State $2r + r\theta = 65$ and $\frac{1}{2}r^2\theta = 225$   | <b>B1</b>  |  | June/<br>9709/1<br>3/<br>2024/<br>Q6 |
|         | Form a 3-term quadratic or cubic in $r$ or $\theta$ or $r\theta$ from correct arc and sector formula                       | <b>*M1</b> | Condone sign errors.   |                                      |
|         | Solve <i>their</i> 3 term quadratic or cubic to obtain values of $r$ or $\theta$   | <b>DM1</b> | Expect $2r^2 - 65r + 450 = (2r - 45)(r - 10)$ or<br>$18\theta^2 - 97\theta + 72 = (9\theta - 8)(2\theta - 9)$ .  |                                      |
|         | $r = 10$ and $\theta = 4.5$ ignore $r = 22.5$ and $\theta = \frac{8}{9}$ , do not ignore $r = 0$                           | <b>A1</b>  | <b>B1 SC</b> if no quadratic or cubic solution.<br>If $r = 0$ included A0 or B0 SC.  |                                      |
|         |  | <b>4</b>   |  |                                      |
| (b)     | Use correct formula for area of triangle with clear use of angle being $2\pi - \text{their } \theta$                       | <b>M1</b>  | Expect 1.783 or $102.2^\circ$ , <i>their</i> $\theta$ must be reflex.  |                                      |
|         | 48.9   | <b>A1</b>  | AWRT, WWW or a second answer.<br>Or greater accuracy; condone absence of units.  |                                      |
|         |  | <b>2</b>   |  |                                      |
| (a)     | $k = \frac{2}{3}$  | <b>B1</b>  | Allow $ACB = \frac{2\pi}{3}$ .   | Nov/9<br>709/11<br>/<br>2023/<br>Q6  |
|         |  | <b>1</b>   |  |                                      |
| (b)     | Perimeter of shaded area = $2\pi r$  | <b>B1</b>  |  |                                      |
|         |  | <b>1</b>   |  |                                      |

|      |   |     |  |                                     |
|------|---|-----|--|-------------------------------------|
| (c)  | Major sector $OAB = \frac{1}{2}r^2 \times \frac{4\pi}{3}$   | *M1 | Expect $\frac{2}{3}\pi r^2$ . Finds area of any relevant sector or triangle. Can be embedded in segment formula.   |                                     |
|      | One or both segments = $[2] \times \left( \frac{1}{2}r^2 \times \frac{\pi}{3} - \frac{1}{2}r^2 \sin \frac{\pi}{3} \right)$                  | *M1 |  |                                     |
|      | = $[2] \left( r^2 \frac{\pi}{6} - r^2 \frac{\sqrt{3}}{4} \right)$   | A1  |  |                                     |
|      | Shaded area = $\frac{2}{3}\pi r^2 - 2 \left( \frac{1}{6}\pi r^2 - \frac{r^2\sqrt{3}}{4} \right)$  | DM1 |  |                                     |
|      | = $\frac{\pi r^2}{3} + \frac{r^2\sqrt{3}}{2}$   | A1  |  |                                     |
| i(a) | [Arc length =] $2 \times \frac{\pi}{3}$ or $\frac{60}{360} \times 2\pi \times 2$  | B1  | Finding one correct arc length – may be implied by correct final answer.   | Nov/9<br>709/12<br>/<br>2023/<br>Q4 |
|      | [Perimeter =] $2\pi$ or 6.28  | B1  | AWRT   |                                     |
| b)   | [Area of one sector =] $\frac{1}{2} \times 2^2 \times \frac{\pi}{3}$ or $\frac{60}{360} \times \pi \times 2^2$ [= $\frac{2\pi}{3}$ or 2.09] | B1  | SOI AWRT   |                                     |
|      | [Area of triangle =] $\frac{1}{2} \times 2^2 \times \sin \left( \frac{\pi}{3} \right)$ or other valid method<br>[= $\sqrt{3}$ or 1.73]      | B1  | AWRT<br>Allow use of $60^\circ$  |                                     |
|      | [Area of coin = 3 segments + triangle $\Rightarrow$ ] $3 \left( \frac{2\pi}{3} - \sqrt{3} \right) + \sqrt{3}$ [= 2.82]                      | M1  | OE<br>Or 3 sectors – 2 triangles $\left( 3 \times \frac{2\pi}{3} - 2 \times \sqrt{3} \right)$ or<br>Sector + 2 segments $\left( \frac{2\pi}{3} + 2 \left( \frac{2\pi}{3} - \sqrt{3} \right) \right)$ |                                     |
|      | $2\pi - 2\sqrt{3}$ or $2(\pi - \sqrt{3})$   | A1  | Must be one of these simplified versions but equivalent decimal answers can score B1B1M1   |                                     |
|      |   | 4   |  |                                     |

|     |   |            |   |                                      |
|-----|---|------------|---|--------------------------------------|
| (a) | Angle $ACO = 0.7$   | <b>BI</b>  | Don't allow AWRT 0.7 .  | Nov/9<br>709/13<br>/<br>2023/<br>Q10 |
|     |   | <b>I</b>   |   |                                      |
| (b) | $[R =] 1.53 r$  | <b>BI</b>  | Allow AWRT 1.53r.   |                                      |
|     |   | <b>I</b>   |   |                                      |
| (c) | Sector $OAB = \frac{1}{2}r^2 \times 2.8 [= 1.4r^2]$   | <b>BI</b>  |   |                                      |
|     | Sector $CAB = \frac{1}{2}(\text{their } R)^2 \times 2 \times \text{their } 0.7$   | <b>*MI</b> |   |                                      |
|     | $1.638r^2$  | <b>AI</b>  | Allow AWRT $1.64r^2$ .  |                                      |
|     | $[2] \times \frac{1}{2}r^2 \sin(\pi - 1.4)$ OR $[2] \times \frac{1}{2}r \times \text{their } R \sin 0.7$  | <b>*MI</b> |   |                                      |
|     | $2 \times 0.4927r^2$  | <b>AI</b>  | Allow AWRT $0.98r^2$ to $0.99r^2$ .   |                                      |
|     | $1.4r^2 - (\text{their } 1.638r^2 - \text{their } 0.985r^2)$  | <b>DMI</b> |   |                                      |
|     | $0.747r^2$ to $0.748r^2$  | <b>AI</b>  |   |                                      |
|     |   | <b>7</b>   |   |                                      |
|     | $\frac{1}{2} \times 8^2 \times \theta = \frac{16\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$   | <b>BI</b>  | SOI OE e.g. $\frac{2\pi}{12}$ , 0.524(3s.f.)<br>Use of degrees acceptable throughout provided conversion used in formulae for sector area and arc length.   | June/9<br>709/11<br>/<br>2023/<br>Q4 |
|     | Arc length = $8 \times \text{their } \frac{\pi}{6} [= 4.1887\dots]$   | <b>MI</b>  | OE FT <i>their</i> $\theta$ . Look for $\frac{4\pi}{3}$ .   |                                      |
|     | $[BC =] 2 \times 8 \sin\left(\frac{1}{2} \times \text{their } \frac{\pi}{6}\right) [= 4.1411\dots]$   | <b>MI</b>  | Attempt to find $BC$ or $BC^2$ (see alt. methods below)<br>FT <i>their</i> $\theta$ . Look for $16\sin\frac{\pi}{12}$ or $4\sqrt{6} - 4\sqrt{2}$ .  |                                      |
|     | Perimeter = 8.33<br><br>ALT 1 $BC^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \cos\left(\text{their } \frac{\pi}{6}\right) [\Rightarrow BC = 4.14\dots]$<br>ALT 2 $BC^2 = (8 - 4\sqrt{3})^2 + 4^2 [\Rightarrow BC = 4.14\dots]$ | <b>AI</b>  | AWRT Must be combined into one term.<br><br>ALT 1 Substitute into correct cosine rule. FT <i>their</i> $\theta$<br>Look for $128 - 64\sqrt{3}$<br><br>ALT 2 Find lengths 4 and $4\sqrt{3}$ then use Pythagoras in the left hand triangle. |                                      |
|     |   |            |   |                                      |
|     | ALT 3 $\frac{BC}{\sin\left(\frac{\pi}{6}\right)} = \frac{8}{\sin\left(\frac{5\pi}{12}\right)} [\Rightarrow BC = 4.14\dots]$   |            | ALT 3 Substitute into correct sine rule.  |                                      |
|     |   | <b>4</b>   |   |                                      |

|     |   |            |   |                                      |                                   |
|-----|---|------------|---|--------------------------------------|-----------------------------------|
| (a) | $\frac{1}{2}OA = x \cos \theta$ or $\frac{OA}{\sin(\pi - 2\theta)} = \frac{x}{\sin \theta}$ or<br>$OA^2 = x^2 + x^2 - 2x^2 \cos(\pi - 2\theta)$ or<br>$x^2 = r^2 + x^2 - 2rx \cos \theta$ or other valid method.        | <b>*B1</b> | Correct expression containing $\frac{1}{2}OA$ , $OA$ or $OA^2$ (allow $p$ , $a$ or $r$ for $OA$ ) containing only terms with $x$ and $\theta$ but not just $OA = 2x \cos \theta$ .<br>Do not condone $\sin \pi - 2\theta$ until missing brackets recovered or $\cos(180 - 2\theta)$ until it becomes $-\cos 2\theta$ etc. | June/9<br>709/12<br>/<br>2023/<br>Q6 |                                   |
|     | $OA = 2x \cos \theta$ leading to Arc length = $2x\theta \cos \theta$  | <b>DB1</b> | AG Complete correct method showing all necessary working. Condone $2x \cos \theta \times \theta$ .  |                                      |                                   |
|     |   | <b>2</b>   | If B0 but www then <b>SCBI</b> for $OA = 2x \cos \theta$ leading to Arc length = $2x\theta \cos \theta$ .   |                                      |                                   |
| (b) | Sector area = $\frac{1}{2}(2x \cos \theta)^2 \times \theta$   | <b>M1</b>  | OE Using sector formula with a correct OA. Condone $\cos^2 \theta$ for $\cos^2 \theta$ and missing brackets.  |                                      |                                   |
|     | Triangle area = $\frac{1}{2} \times 2x \cos \theta \times x \sin \theta$ OR $\frac{1}{2}x^2 \sin(\pi - 2\theta)$  | <b>M1</b>  | Using a correct triangle formula for the correct triangle. Condone missing brackets and 180 for $\pi$ .   |                                      |                                   |
|     | [Area APB =] Their sector area – their triangle area  | <b>M1</b>  | Both expressions must be areas involving terms with $x^2$ and $\theta$ only. Condone missing brackets and 180 for $\pi$ for the triangle. Condone calling the sector a segment.   |                                      |                                   |
|     | [Area APB =] $\frac{1}{2}(2x \cos \theta)^2 \times \theta - \frac{1}{2}x^2 \sin(\pi - 2\theta)$<br>[= $x^2(2\theta \cos^2 \theta - \frac{1}{2} \sin 2\theta)$ or $x^2 \cos \theta(2\theta \cos \theta - \sin \theta)$ ] | <b>A1</b>  | OE<br>A correct expression. Mark the first unsimplified result of subtraction and ISW any incorrect 'simplifications'.  |                                      |                                   |
|     | <b>4</b>  |            |   |                                      |                                   |
| (a) | $\tan BDC = \frac{4}{3}$ or $\sin BDC = \frac{4}{5}$ or $\cos BDC = \frac{3}{5}$ used to find ADC   | <b>M1</b>  | May use cosine rule or $CAD = \tan^{-1} \frac{4}{8}$ .  |                                      | March<br>/9709/<br>12/202<br>3/Q8 |
|     | $BDC = 0.927[3] \rightarrow ADC = \pi - 0.927[3] [= 2.214 \text{ to } 2.215]$   | <b>A1</b>  | Allow degrees, 126.87, and $0.7048\pi$ or $0.705\pi$  |                                      |                                   |
|     | $Arc AC = 5 \times \text{their } 2.214$   | <b>M1</b>  | Use of $r\theta$ or $\frac{\theta}{360} \cdot 2\pi r$ Expect 11.07.   |                                      |                                   |
|     | $AC = \sqrt{8^2 + 4^2}$ or $2 \times 5 \times \sin 1.107$   | <b>M1</b>  | Expect 8.94.  |                                      |                                   |
|     | [Perimeter = $11.07 + 8.94 = 20.0$ ]  | <b>A1</b>  | Accept AWRT [20.01, 20.02].   |                                      |                                   |
|     |   | <b>5</b>   |   |                                      |                                   |
| (b) | Sector $ACD = \frac{1}{2} \times 5^2 \times \text{their } 2.214$  | <b>M1</b>  | See use of $\frac{1}{2}r^2\theta$ or $\frac{\theta}{360} \cdot \pi r^2$ . Expect 27.7.  |                                      |                                   |
|     | Subtracting the area of $\triangle ADC = \frac{1}{2} \times 5 \times 4$ or $\frac{1}{2}5^2 \sin \text{their } 2.214$ or<br>$\frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 3 \times 4$                              | <b>M1</b>  | Subtracting the area of $\triangle ADC$ , expect -10.   |                                      |                                   |
|     | Shaded area = $27.7 - 10 = 17.7$  | <b>A1</b>  | Accept AWRT [17.67, 17.68]. Correct answer cannot come from an angle of 2.215.  |                                      |                                   |
|     |   | <b>3</b>   |   |                                      |                                   |