(a)	Attempt correct process for solving 3-term quadratic equation in $\sqrt{x}$	M1	Accept $8y^2 - 6y - 9 \to (2y-3)(4y+3)$, if $y = \sqrt{x}$ specified.	Nov/11/2024/Q5
	Obtain at least $2\sqrt{x} - 3 = 0$ or equivalent	A1	Ignore $4\sqrt{x} + 3 = 0$. **SC B1** for $\sqrt{x} = \frac{3}{2}$ with no method shown for solving the 3-term quadratic.	
	Conclude $x = \frac{9}{4}$ ignore $\frac{9}{16}$	A1	**SC B1** if no method shown for solving the 3-term quadratic.	
	Alternative Method for Q5(a)			
	$3\sqrt{x} = 4x - \frac{9}{2} \to 16x^2 - 45x + \frac{81}{4}$ o.e and attempt correct process to solve	M1		
	Obtain $x = \frac{9}{4}$ or $\frac{9}{16}$	A1	**SC B1** if no method shown for solving the 3-term quadratic.	
	$x = \frac{9}{4}$ ignore $\frac{9}{16}$	A1	**SC B1** if no method shown for solving the 3-term quadratic.	
		3		
(b)	Integrate to obtain form $k_1x^2 + k_2x^{\frac{3}{2}} + k_3x$ where $k_1k_2k_3 \neq 0$	M1		
	Obtain correct $2x^2 - 2x^{\frac{3}{2}} + x$ or equivalent	A1	Allow unsimplified.	
	Substitute $x = 4$ and $y = 11$ to attempt value of c	M1	Dependent on at least 2 correct terms involving x.	
	Obtain $y = 2x^2 - 2x^{\frac{3}{2}} + x - 9$	A1	Must be simplified. Allow 'f(x) ='. Allow y missing if y appears previously.	
		4		
(a)	State or imply centre of C_1 is $(-3, 5)$	B1		Nov/11/2024/Q6
	State or imply centre of C_2 is $(9, -4)$	B1		
	Attempt correct process for finding distance between centres	M1		
	Obtain 15	A1		
		4		
(b)	$R = 4$ and $R = 8$	B1		
	Obtain least or greatest distance	B1 FT	'15' – R_1 – R_2 or '15' + R_1 + R_2.	
	Obtain 3 and 27	B1 FT	'15' – R_1 – R_2 and '15' + R_1 + R_2.	
		3		
(a)	Differentiate to obtain form $k_1(2x+1)^{-\frac{4}{3}}$	M1		Nov/11/2024/Q7
	Obtain correct $-8(2x+1)^{-\frac{4}{3}}$ or unsimplified equivalent	A1		
	Attempt equation of tangent at $\left(\frac{7}{2}, 6\right)$ with numerical gradient	M1	Gradient must come from a differentiated expression.	
	Obtain $y = -\frac{1}{2}x + \frac{31}{4}$ or equivalent of requested form	A1		
		4		

(b)	Integrate to obtain form $k_2(2x+1)^{\frac{2}{3}}$	M1		
	Obtain correct $9(2x+1)^{\frac{2}{3}}$ or unsimplified equivalent	A1		
	Use correct limits correctly to find area	M1	Substitute correct limits into an integrated expression. 36 – 9 minimum working required.	
	Obtain 27	A1	SC B1 if M1 A1 M0 scored.	
		4		
(a)	$\left[f(2+h)=\right]\ 2(2+h)^2-3$	B1	SOI	Nov/12/2024/Q3
	$\dfrac{\left(2(2+h)^2-3\right)-5}{(2+h)-2}\quad\left[=\dfrac{2h^2+8h}{h}\right]$	M1	$\dfrac{\left\{their\left(2(2+h)^2-3\right)\right\}-their5}{(2+h)-2}$ can be implied by the simplified expression or the correct answer. *Their* 5 must come from $2(2)^2-3$.	
	$2h+8$ or $2(h+4)$	A1		
		3		
(b)	$h\to 0$, or chord [AB] $\to$ tangent [at A]	B1	Either of these statements or any sight of $h=0$.	
	8	B1FT	Could come from anywhere except wrong working. Either correct or FT their linear expression from **(a)**.	
		2		
(a)	$\left(x-\left(-\frac{1}{2}p\right)\right)^2+\left(y-(-1)\right)^2$ OE	B1*	Allow $a=-\frac{1}{2}p$ and $b=-1$, or centre is $\left(-\frac{1}{2}p,-1\right)$.	Nov/12/2024/Q8
	$\left(x-\left(-\frac{1}{2}p\right)\right)^2+\left(y-(-1)\right)^2=-q+1+\left(-\frac{1}{2}P\right)^2$ OE	DB1		
		2		
b)(i)	[Gradient of tangent =] $-\frac{1}{2}$	B1	OE SOI	
	[Gradient of normal =] 2	M1	Use of $m_1m_2=-1$ with *their* numeric tangent gradient.	
	$\dfrac{y-3}{x-4}=2\ \left[y=2x-5\right]$	A1	OE ISW Allow $y=2x+c,\ 3=2\times4+c\ \Rightarrow c=-5$.	
		3		

(b)(ii)	**Method 1 for the first two marks:**			
	$-1-3=2\left(-\frac{1}{2}p-4\right)$ or $-1=-p-5$	**M1***	Using *their* stated centre or $\left(\frac{\pm p}{2},\ \pm 1\right)$ in *their* equation of the normal.	
	$p=-4$	**A1**		
	Method 2 for the first two marks:			
	$-1=2x-5 \Rightarrow x=2 \Rightarrow -\frac{1}{2}p=2$	**M1***	Using their normal equation and *their* stated centre or $\left(\frac{\pm p}{2},\ \pm 1\right)$.	
	$p=-4$	**A1**		
	Method 3 for the first two marks:			
	$2x+2y\frac{dy}{dx}+p+2\frac{dy}{dx}=0$ $\left[\Rightarrow p=-8-8\frac{dy}{dx}\right]$	**M1***		
	$\left[\frac{dy}{dx}=-\frac{1}{2}\Rightarrow\right]$ $p=-4$	**A1**		
(b)(ii)	**Method 1 for the last 3 marks:**			
	$r^2=(4-2)^2+(3-(-1))^2$ $[=20]$	**M1***	Using (4, 3) and *their* centre or $\left(\frac{\pm their\ p}{2}, \pm 1\right)$ to find r^2 or r.	
	$-q+1+\frac{1}{4}p^2=20$	**DM1**	OE Using *their* expression for r^2 (from **(a)**) equated to *their* 20.	
	$q=-15$	**A1**		
	Method 2 for the last 3 marks:			
	$r=\frac{\lvert 2-2-10\rvert}{\sqrt{5}}$ $\left[=\frac{10}{\sqrt{5}}\right]$	**M1***	Using $(2,-1)$ and $x+2y-10=0$ (distance from a point to a line).	
	$-q+1+\frac{1}{4}p^2=\left(\frac{10}{\sqrt{5}}\right)^2$	**DM1**	OE Using *their* expression for r^2 equated to *their* $\left(\frac{10}{\sqrt{5}}\right)^2$.	
	$q=-15$	**A1**		
	Method 3 for the last 3 marks:			
	$4^2+3^2+4p+6+q=0$ $[\Rightarrow 4p+q+31=0]$ OR $\left(4-\left(-\frac{1}{2}p\right)\right)^2+(3-(-1))^2=-q+1+\left(-\frac{1}{2}p\right)^2$	**M1***	Substituting $(4,3)$ into their circle equation.	
	$4(-4)+q+31=0$	**DM1**	Substituting *their* $p=-4$.	
	$q=-15$	**A1**		

8(b)(ii)	**Alternative Method for Question 8(b)(ii)**			
	$4^2+3^2+4p+6+q=0$ $x^2+(2x-5)^2+px+2(2x-5)+q=0$ with $x=4$ $x^2+\left(\frac{10-x}{2}\right)^2+px+2\left(\frac{10-x}{2}\right)+q=0$ with $x=4$ $\left(\frac{y+5}{2}\right)^2+y^2+p\left(\frac{y+5}{2}\right)+2y+q=0$ with $y=3$ $(10-2y)^2+y^2+p(10-2y)+2y+q=0$ with $y=3$ {Each of these $\Rightarrow$ $4p+q+31=0$}	M1*	Substituting $(4, 3)$ into *their* circle equation, or replacing y with $2x-5$ from the normal equation, or replacing y with $\frac{10-x}{2}$ from the tangent equation, or replacing x with $\frac{y+5}{2}$ from the normal equation, or replacing x with $10-2y$ from the tangent equation, and using either $x=4$ or $y=3$ to form an equation in p and q.	
	$\frac{5}{4}x^2+(p-6)x+35+q=0 \Rightarrow (p-6)^2-4\times\frac{5}{4}\times(35+q)=0$ OR $5y^2-y(38+2p)+100+10p+q=0 \Rightarrow (38+2p)^2-4\times5\times(100+10p+q)=0$ {Each of these $\Rightarrow p^2-12p-139-5q=0$}	M1*	Solving the tangent and circle equations simultaneously to form a quadratic equation in either x or y. Then using $b^2-4ac=0$ on their quadratic to form an equation in p and q.	
	Solving the equations simultaneously to find p or q	DM1		
	$p=-4$	A1		
	$q=-15$	A1		
		5		

(a)	Gradient of $AB=\frac{-5-3}{8-4}$ $[=-2]$	M1*		Nov/13/2024/Q10
	Midpoint $AB=\left(\frac{8+4}{2},\frac{-5+3}{2}\right)$ $[(6,-1)]$	M1		
	Gradient of normal $=-\frac{1}{-2}$ $\left[=\frac{1}{2}\right]$ and an attempt to find the required equation	DM1	Must be used to find equation of perpendicular through *their* $(6,-1)$.	
	Equation of perpendicular bisector is $y+1=\frac{1}{2}(x-6)$, so $y=\frac{1}{2}x-4$	A1	WWW AG – working involving the perpendicular bisector must be seen.	
	Alternative Method for Question 10(a)			
	$AC^2=(a-4)^2+(b-3)^2$, $BC^2=(a-8)^2+(b+5)^2$ both expanded	M1*		
	Solving $AC=BC$ [= 10]	DM1	Only allow a single sign error.	
	Eliminating a^2 and b^2	DM1	May be awarded before the previous DM1.	
	$a=2b+8$, concluding $y=\frac{x}{2}-4$	A1	WWW	
		4		

(b)	Using the centre as $\left(a,\ \frac{1}{2}a-4\right)$	M1	May see centre as $(2y+8, y)$ OE. May be seen in an incorrect equation.	
	$(4-a)^2+(3-0.5a+4)^2=100$	M1	Sub in (4, 3) or (8, −5). Could use circle with $(6,-1)$ and $r=\sqrt{80}$.	
	$1.25a^2-15a-35\ [=0] \Rightarrow a^2-12a-28\ [=0]\ \left(\text{or } b^2+2b-15\ [=0]\right)$	DM1	Obtain a 3-term quadratic in *their* x or y.	
	$[(a-14)(a+2)=0] \Rightarrow a=14,\ a=-2$	A1	Or $[(b-3)(b+5)=[0]] \Rightarrow b=3,\ b=-5$.	
	$\Rightarrow (x-14)^2+(y-3)^2=100$ and $(x+2)^2+(y+5)^2=100$	A1		
	Alternative Method 1 for the first 3 marks:			
	Make a or b the subject from a circle centre (a,b) using A or B	M1	E.g. $b=\sqrt{100-(y-3)^2}+4$ from circle through A. These equations may have been found in part **(a)**.	
	Form an equation in a or b only	M1	Substitute *their* a or b into their second circle equation.	
	Simplify to a quadratic in a or b	DM1	Expect $a^2-12a-28=0$ or $b^2+2b-15=0$, OE.	
	Alternative Method 2 for the first 3 marks:			
	Obtaining CM (C, centre; M, mid-point of AB)	M1	Expect $\sqrt{80}$. Must be clear this is CM, not AB.	
	Using the triangle CMT, where CT is parallel to the x-axis, to find the vertical distance of C from M, MT	DM1	Expect $MT=4$.	
	Using the triangle CMT, where MT is parallel to the y-axis, to find the horizontal distance of C from M, CT	DM1	Expect $CT=8$.	
		5		

$(x-3)^2+y^2=18\quad y=mx-9$ leading to $(x-3)^2+(mx-9)^2=18$	M1	Finding equation of tangent and substituting into circle equation. Must be $mx-9$.	June/11/2024/Q10
$x^2-6x+9+m^2x^2-18mx+81=18$ leading to $(m^2+1)x^2-(6+18m)x+72[=0]$	M1	Brackets expanded and all terms collected on one side of the equation. May be implied in the discriminant. m cannot be numeric.	
$(6+18m)^2-4(m^2+1)\times 72[=0]$	*M1	Use of b^2-4ac. Not in quadratic formula. m cannot be numeric, c must be numeric.	
$36m^2+216m-252[=0]$ [leading to $m^2+6m-7=0$]	DM1	Simplifies to 3 term quadratic.	
$m=1$ or $m=-7$	A1	Condone no method for solving quadratic shown.	
$m=1$ leading to $2x^2-24x+72=0$ leading to $x=6$	DM1	Must be correct x for *their* quadratic.	
$m=-7$ leading to $50x^2+120x+72=0$ leading to $x=-\frac{6}{5}$	DM1	Must be correct x for *their* quadratic.	
$(6,-3),\left(-\frac{6}{5},-\frac{3}{5}\right)$	A1		

Alternative Method 1 for first 4 marks of Question 10			
$\dfrac{\lvert 3m-1(0)-9\rvert}{\sqrt{m^2+1}}$	(M1)	Use of the formula for the length of a perpendicular from a point to a line.	
$\dfrac{\lvert 3m-1(0)-9\rvert}{\sqrt{m^2+1}} = \sqrt{18}$	(M1)	Equates length of a perpendicular from a point to a line to the radius.	
$(3m-9)^2 = 18(m^2+1)$	(M1)	Squares and clears the fraction.	
$9m^2 - 54m + 81 = 0$ $\left[\text{leading to } m^2+6m-7=0\right]$	(M1)		
Alternative Method 2 for first 3 marks of Question 10			
$(3-x)(9+6x-x^2)^{-1/2} = m$	(M1)	OE Differentiates implicitly or otherwise and equates $\dfrac{dy}{dx}$ to m.	
$(1+m^2)x^2 - 6(1+m^2)x + 9(1-m^2)[=0]$	(M1)	Brackets expanded and all terms collected on one side of the equation. May be implied in the discriminant.	
$36(1+m^2)^2 - 4(1+m^2)\times 9(1-m^2)[=0]$	(M1)	Use of b^2-4ac.	
	8		

(a)	$(x-6)^2 + (2a-x+a)^2 = 18$	M1*	Replacing y with $2a-x$ in the circle equation, condone incorrect expansion before substitution.	June/ 12/ 2024 /Q7
	$2x^2-12x-6ax+9a^2+36-18[=0]$	A1	All terms collected on one side of the equation. May be implied by the discriminant.	
	$(12+6a)^2 - 4\times 2\times(9a^2+18)\,[=0]$	DM1	Correct use of "$b^2 - 4ac$" from *their* 3 term quadratic equation in x, with an x term of the form $(m+na)x$ with both m and $n \neq 0$.	
	$-36a^2+144a[+0=0]$	A1		
	$a=0,\ a=4$	A1		
		5		
(b)	[Centre is] (6, −4) or [Point of intersection is] (9, −1)	B1		
	[Gradient of diameter] $=1$	B1		
	$y+4=x-6$ or $y+1=x-9$ $\quad$ [leading to $y=x-10$]	B1FT	FT on *their* point of intersection or *their* centre with an x co-ordinate of ±6 and gradient = 1.	
		3		

Alternative Method 2: for the last 5 marks			June/ 13/ 2024 /Q8
$\widehat{ACP} = \widehat{MAP} = \tan^{-1}\frac{4}{3}$ or identifying similar triangles *PMA* and *AMC*	(M1A1)	*C* is the circle centre, *P* is intersection of the two tangents, *M* is intersection of *PC* and the *y*-axis.	
$\tan MAP = \frac{PM}{4}, \frac{4}{3} = \frac{PM}{4}, PM = \frac{16}{3}$ or use of similar triangles	(M1A1)		
P is $\left(\frac{-16}{3}, -1\right)$	(A1)		
Alternative Method 3: for the last 5 marks			
Pythagoras on triangle *PAC*, $PC^2 = PA^2 + AC^2$,	(M1)	Identifies the required 3 sides and sets up formula.	
$PC^2 = (PM+3)^2$, $PA^2 = PM^2 + 4^2$, $AC = \text{radius} = 5$	(A1)	Finds each side with two in terms of *PM* OE.	
$(PM+3)^2 = PM^2 + 4^2 + 5^2$ leads to $6PM = 32$, $PM = \frac{16}{3}$	(M1A1)	Sets up and solves equation.	
P is $\left(\frac{-16}{3}, -1\right)$	(A1)		
	8		

Differentiate to obtain form $kx(2x^2-5)^{-2}$	M1		March/ 12/ 2024 /Q5
Obtain correct $-12x(2x^2-5)^{-2}$	A1	OE	
Substitute (2, 1) to obtain gradient $-\frac{24}{9}$	A1	OE e.g. $-\frac{8}{3}$. Allow -2.67.	
Apply negative reciprocal to *their* numerical gradient to obtain gradient of normal	*M1	Must have been some attempt at differentiation. Expect $\frac{3}{8}$	
Attempt equation of normal using *their* gradient of the normal and (2, 1)	DM1	Expect $y-1=\frac{3}{8}(x-2)$.	
Obtain $3x-8y+2=0$ (allow multiples)	A1	Or equivalent of requested form e.g. $8y-3x-2=0$.	
	6		

(a)	Attempt substitution for *y* in quadratic equation	*M1	Or substitution for *x* …	March/ 12/ 2024 /Q7
	Obtain $5x^2+30x+75-k[=0]$ or $5y^2-20y+50-k[=0]$	A1	OE e.g. $x^2+6x+15-\frac{k}{5}$ (all terms gathered together).	
	Use $b^2-4ac=0$ with *their a*, *b* and *c*	DM1	'= 0' may be implied in subsequent working or the answer.	
	Obtain $900-20(75-k)=0$ or equivalent and hence $k=30$	A1	… obtaining $400-20(50-k)=0$ and $k=30$.	
		4		
(b)	Substitute *their* value of *k* in equation from part **(a)** and attempt solution	M1	Expect $5x^2+30x+45[=0]$ or $5y^2-20y+20[=0]$.	
	Obtain coordinates $(-3,\ 2)$	A1	**SC B1** only $(-3,\ 2)$ without attempt at quadratic solution.	
		2		

)(a)	Obtain gradient of relevant radius is –2	B1		March/12/2024/Q10
	Using $m_1 m_2 = -1$ obtain the gradient of the tangent and use it to form a straight line equation for a line containing (–6, 9)	M1	m_1 must be from an attempt to find the gradient of the radius using the centre and the given point.	
	Obtain $y = \frac{1}{2}x + 12$	A1	OE e.g. $y - 9 = \frac{1}{2}(x+6)$.	
		3		
)(b)	State or imply $(x+4)^2 + (y-5)^2 = 20$	B1	If $x^2 + y^2 - 2gx - 2fy + c = 0$ is used correctly with $(-g, -f) = (-4, 5)$ and $c = g^2 + f^2 - r^2$ then M1.	
	Obtain $x^2 + y^2 + 8x - 10y + 21 = 0$	B1	A1 if above method used.	
		2		
)(c)	Substitute $x = 0$ in equation of circle to find y-values 3 and 7 or state C to AB = 4	B1	May be implied by AB = 4 or use of |x-coordinate of C|.	
	Attempt value of θ either using cosine rule or via $\frac{1}{2}\theta$ using right-angled triangle	M1	Using *their* AB. If $\theta/2$ used, must be multiplied by 2.	
	Obtain $\theta = 0.9273$	A1	Or greater accuracy. A correct answer implies the M1.	
		3		
)(d)	Attempt arc length using $r\theta$ formula with *their* θ (not *their* $\theta/2$) and $r = \sqrt{20}$	M1	Expect 4.15.	
	Obtain perimeter = 8.15 or greater accuracy	A1	Condone missing units or incorrect units.	
	Attempt area using $\frac{1}{2}r^2(\theta - \sin\theta)$ formula or equivalent with *their* θ and $r = \sqrt{20}$	M1	If sector – triangle used, both formulae must be correct. If triangle ACM used, area must be multiplied by 2.	
	Obtain area = 1.27 or greater accuracy	A1	Condone missing units or incorrect units.	
		4		