Revision – Unit1 Quadratics (5types)

1- Completing the square $(a \pm b)^2 = a^2 \pm 2ab + b^2$ ✓

(a) Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. **[2]**

Nov 2024 /13/Q8

$$\boxed{3(x-2)^2 + 2}\ 2 ✓$$

$$3x^2 - 12x - 14$$

$$3(x^2 - 4x + 4 - 4) + 14$$

$$3(x^2 - 4x + 4) - 12 + 14$$

$$3(x-2)^2 + 2 ✓$$

$$3x^2 - 12x + 14 = 3(x+a)^2 + b$$
$$= 3(x^2 + 2ax + a^2) + b$$
$$3x^2 - 12x + 14 = 3x^2 + 6ax + 3a^2 + b$$

$$6a = -12 \qquad 3a^2 + b = 14$$
$$a = -2 \qquad b = 14 - 12 = 2$$

(a) By expressing $-2x^2 + 8x + 11$ in the form $-a(x-b)^2 + c$, where a, b and c are positive integers, find the coordinates of the vertex of the graph with equation $y = -2x^2 + 8x + 11$. **[3]**

Nov 2024 /12/Q7

$$a(x+p)^2 + q$$

$$(-p, q)$$

$$(-(-b), c)$$

$$(b, c)$$

(a) Express $3y^2 - 12y - 15$ in the form $3(y+a)^2 + b$, where a and b are constants. [2]

(a) Express $4x^2 - 24x + p$ in the form $a(x+b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. [2]

(a) Express $x^2 - 8x + 11$ in the form $(x+p)^2 + q$ where p and q are constants. [2]

June2022/11/Q1

(a) Express $2x^2 - 8x + 14$ in the form $2[(x-a)^2 + b]$. [2]

March 2022 /12/Q5

$$2\left[x^2 - 4x + 7\right)$$

(a) Express $-3x^2 + 12x + 2$ in the form $-3(x-a)^2 + b$, where a and b are constants. [2]

(a) Express $5y^2 - 30y + 50$ in the form $5(y+a)^2 + b$, where a and b are constants. [2]

(a) Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

2- The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(b) Hence find the exact solutions of the equation $3x^4 - 12x^2 - 15 = 0$. [3]

Put $y = x^2$

$3y^2 - 12y - 15 = 0$ ✓ 1M

$y = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 3 \times (-15)}}{2 \times 3}$ 1M
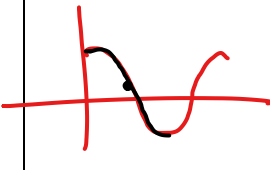
$y = 5$, $y = -1$

$x^2 = 5$, $x^2 = -1$ ✗

$x = \pm\sqrt{5}$ — 1A

(b) Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$. [2]

$x^2 - 8x + 16 = 0$

3- Solving more complex quadratic equations

↓

Solve the equation $8x^6 + 215x^3 - 27 = 0$. [3]

Put $y = x^3$

$8y^2 + 215y - 27 = 0$

$y = $ ———

$y = \dfrac{1}{8}$, $y = -27$

$x = \dfrac{1}{2}$ $x^3 = \dfrac{1}{8}$ $y \cdot x^3 = -27$

$x = -3$

(b) Solve the equation $8\cos^2\theta - 10\cos\theta + 2 = 0$ for $0° \leqslant \theta \leqslant 180°$. [3]

Put $y = \cos\theta$

$8y^2 - 10y + 2 = 0$

$y = 1$, $\dfrac{1}{4}$

$\cos\theta = 1$ $\cos\theta = \dfrac{1}{4}$

$\theta = \cos^{-1}(1)$ $75.5°$

$\theta = 0$

(a) Solve the equation $6\sqrt{y} + \dfrac{2}{\sqrt{y}} - 7 = 0.$ **[4]**

$$6y + 2 - 7\sqrt{y} = 0$$

$$6y - 7\sqrt{y} + 2 = 0$$

put $x = \sqrt{y}$

$$6x^2 - 7x + 2 = 0$$

$$\rightarrow y = \frac{1}{4}, \text{ or } y = \frac{4}{9}$$

(b) Hence solve the equation $6\sqrt{\tan x} + \dfrac{2}{\sqrt{\tan x}} - 7 = 0$ for $0° \leqslant x \leqslant 360°.$ **[3]**

$$\tan x = \frac{1}{4} \qquad , \quad \tan x = \frac{4}{9}$$

$$x = \tan^{-1}\left(\frac{1}{4}\right)$$

$$x = \underline{\quad\quad} \qquad\qquad + 180$$

$b^2-4ac > 0$

2 real and distinct

$b^2-4ac = 0$

2 real and equal roots

$b^2-4ac < 0$

no real roots.

4-The number of roots of a quadratic equation

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

$a = 4$

$b = -24$

$c = p$

$b^2 - 4ac < 0$

$(-24)^2 - 4 \times 4 \times p < 0$

$576 - 16p < 0$

$576 < 16p$

$\boxed{p > 36}$

(a) Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

$ax^2 + bx + c = 0$

$b^2 - 4ac < 0$

$k^2 - 4 \times 8 \times 2 < 0$

$k^2 - 64 < 0$

$-8 < k < 8$

$$16x^2 - 24k + 10 - k = 0$$

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

$$b^2 - 4ac = 0$$

$$(-24)^2 - 4 \times 16 \times (10 - k) = 0$$

5- Intersection of a line and a quadratic curve

Show that the curve with equation $x^2 - 3xy - 40 = 0$ and the line with equation $3x + y + k = 0$ meet for all values of the constant k. [5]

$$y = -3x + k$$

$$x^2 - 3x(-3x - k) - 40 = 0$$

$$x^2 + 9x^2 + 3kx - 40 = 0$$

$$10x^2 + 3k\,x - 40 = 0$$

$$b^2 - 4ac > 0$$

The straight line $y = x + 5$ meets the curve $2x^2 + 3y^2 = k$ at a single point P.

(a) Find the value of the constant k. [4]

$b^2 - 4ac = 0$

(b) Find the coordinates of P. [2]

A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

Show that the line and the curve meet for all values of k. [4]

A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points. [5]

A curve has equation $y = kx^2 + 2x - k$ and a line has equation $y = kx - 2$, where k is a constant.

Find the set of values of k for which the curve and line do not intersect. [5]

A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

Find the set of values of k for which the line and curve have two distinct points of intersection.   [5]