Revision – Quadratics

Completing the square

$3(x-2)^2+2$ or $a=-2,\ b=2$	B1 B1			Nov 2024 /13/Q8
	2			
$-2\left((x\pm p)^2\pm q\right)$ or $-2(x\pm p)^2\pm q$	M1*	$p\neq 0$.		Nov 2024 /12/Q7
$-2\left((x-2)^2\pm q\right)$ or $-2(x-2)^2\pm q$	DM1			
$-2(x-2)^2+19$ and (2, 19)	A1	Accept $x=2, y=19$ or 2, 19.		
	3			
(a) $3(y-2)^2-27$ or $a=-2,\ b=-27$	B1 B1			June2024 /11/Q1
	2			
$4(x-3)^2$ seen or $a=4$ and $b=-3$	B1	OE Award marks for the correct expression or their values a, b and c. Condone $4(x-3)+p-36=0$ and $4\left(\frac{p}{4}-9\right)$.		June 2023 /12/Q3
$-36+p$ or $p-36$ seen or $c=p-36$	B1			
	2			
$p-36>0$ leading to $p>36$ or $24^2-4\times4p\langle0\Rightarrow p\rangle36$ or $36<p$	B1	Allow $(36,\infty)$ or $36<p<\infty$. Consider final answer only.		
	1			
(a) $x^2-8x+11=(x-4)^2\ldots$ or $p=-4$	B1	If p and q-values given after *their* completed square expression, mark the expression and ISW.		June2022 /11/Q1
... -5 or $q=-5$	B1			
	2			
(b) $(x-4)^2-5=1$ so $(x-4)^2=6$ so $x-4=[\pm]\sqrt{6}$	M1	Using *their* p and q values or by quadratic formula		
$x=4\pm\sqrt{6}$ or $\frac{8\pm\sqrt{24}}{2}$	A1	Or **exact** equivalent. No FT; must have ± for this mark. ISW decimals 1.55, 6.45 if exact answers seen. If M0, **SC B1** possible for correct answers.		
	2			
(a) $2[\{(x-2)^2\}\ \{+3\}]$	B1 B1	B1 for $a=2$, B1 for $b=3$. $2(x-2)^2+6$ gains B1B0		March 2022 /12/Q5
	2			
(a) $\{-3(x-2)^2\}\quad\{+14\}$	B1 B1	B1 for each correct term; condone $a=2$, $b=14$.		Nov2021 /11/Q8
	2			

(a)	$\{5(y-3)^2\}$ $\{+5\}$		**B1 B1**	Accept $a=-3$, $b=5$
			2	

Nov 2021 /13/Q3

(a)	$(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$	B1	$k(4x-3)^2$ where $k \neq 1$ scores B0 but mark final answer, allow recovery.
	$+1$ or $b=1$	B1	
		2	
(b)	[For one root] $k=1$ or '*their b*'	B1 FT	Either by inspection or solving or from $24^2-4\times16\times(10-k)=0$ WWW
	[Root or $x=$]$\frac{3}{4}$ or 0.75	B1	**SC B2** for correct final answer WWW.
		2	

June 2021 /12/Q1

The quadratic formula

$(x^2-2)^2=9$ leading to $x^2-2=\pm3$	**M1**	Must be x^2 unless substitution is clear.
$x^2=-1$ or $x^2=5$	**M1**	Allow omission of **-1** if ±3 seen.
$x=\pm\sqrt{5}$	**A1**	**B1 SC** if M1M1 not awarded. Ignore ± i, i, –i, √–1. Use of calculator with no working scores 0/3.
Alternative method for Question 1(b)		
$3x^4-12x^2-15=0$ leading to $3(x^2-5)(x^2+1)[=0]$	**(M1)**	
$x^2=-1$ or $x^2=5$	**(M1)**	Allow omission of –1 if factors seen. Factorising or other valid method.
$x=\pm\sqrt{5}$	**(A1)**	**B1 SC** if M1M1 not scored. Ignore ± i, i, –i, √–1. Use of calculator with no working scores 0/3.
	3	

June2024 /11/Q1

(a)	$x^2-8x+11=(x-4)^2\ldots$ or $p=-4$	**B1**	If p and q-values given after *their* completed square expression, mark the expression and ISW.
	$\ldots -5$ or $q=-5$	**B1**	
		2	
(b)	$(x-4)^2-5=1$ so $(x-4)^2=6$ so $x-4=[\pm]\sqrt{6}$	**M1**	Using *their* p and q values or by quadratic formula
	$x=4\pm\sqrt{6}$ or $\frac{8\pm\sqrt{24}}{2}$	**A1**	Or **exact** equivalent. No FT; must have ± for this mark. ISW decimals 1.55, 6.45 if exact answers seen. If M0, **SC B1** possible for correct answers.
		2	

June 2022 /11/Q1

Solving more complex quadratic equations

$[8x^6+215x^3-27=0]$ leading to $(8x^3-1)(x^3+27)[=0]$ **OR** $\frac{-215\pm\sqrt{215^2-4.8.-27}}{2.8}$ or $\frac{-215\pm\sqrt{47089}}{2.8}$	**M1**	OE If a substitution is used then the correct coefficients must be retained. Condone substitution of $x=x^3$.	June2023 /12/Q4
$\frac{1}{8}$, -27	**A1**	Both correct values seen. **SC:** if M0 scored **SC B1** is available for sight of $\frac{1}{8}$ and -27 OE	
$\frac{1}{2}$ or 0.5, –3	**A1**	**SC:** if M0SCB1 scored then **SCB1** is available for the correct answers and no others. Do not ISW if answers given as a range.	
	3		

			Nov2022 /12/Q3

$x^2-4x+3=mx-6$ leading to $x^2-x(4+m)+9$	***M1**	Equating and gathering terms. May be implied on the next line.	June2022 /13/Q4
b^2-4ac leading to $(4+m)^2-4\times9$	**DM1**	SOI. Use of the discriminant with *their* a, b and c	
$4+m=\pm6$ or $(m-2)(m+10)=0$ leading to $m=2$ or -10	**A1**	Must come from $b^2-4ac=0$ SOI	
Substitute both *their m* values into *their* equation in line 1	**DM1**		
$m=2$ leading to $x=3$; $m=-10$ leading to $x=-3$	**A1**		
(3, 0), (–3, 24)	**A1**	Accept 'when $x=3$, $y=0$; when $x=-3$, $y=24$' If final A0A0 scored, **SC B1** for one point correct WWW	

(a)	$6y+2-7y^{1/2}$ [= 0]	***M1**	OE Rearrange to a 3-term quadratic.	June2022 /13/Q5
	$\left(2y^{\frac{1}{2}}-1\right)\left(3y^{\frac{1}{2}}-2\right)$ [= 0] or e.g. $(2u-1)(3u-2)$ [= 0]	**DM1**	Or use of formula or completing the square.	
	$[y^{1/2}=]\frac{1}{2},\frac{2}{3}$	**A1**	Answers only **SC B1** if DM1 not scored.	
	$[y=]\frac{1}{4},\frac{4}{9}$	**A1**	Answers only **SC B1** if DM1 not scored.	
		4		
(b)	Use of $\tan x$ = *their y* values	**M1**	Must have at least 2 values of y from part **(a)**.	
	$x=$ 14[.0], 24[.0], $x=$ 194[.0], 204[.0]	**A1** **A1 FT**	FT for 180 + angle (twice). AWRT	
		3		

The number of roots of a quadratic equation

$4(x-3)^2$ seen or $a=4$ and $b=-3$	B1	OE Award marks for the correct expression or their values a, b and c. Condone $4(x-3)+p-36=0$ and $4\left(\frac{p}{4}-9\right)$.	June2023 /12/Q3
$-36+p$ or $p-36$ seen or $c=p-36$	B1		
	2		
$p-36>0$ leading to $p>36$ or $24^2-4\times4p\{0\Rightarrow p\}36$ or $36<p$	B1	Allow $(36,\infty)$ or $36<p<\infty$. Consider final answer only.	
	1		

$k^2-4\times8\times2$ [<0]	M1	Use of b^2-4ac but not just in the quadratic formula.	Nov2022 /12/Q3
$-8<k<8$ or $-8<k$, $k<8$ or $\lvert k\rvert<8$ or $(-8, 8)$	A1	Condone '$-8<k$ or $k<8$', '$-8<k$ and $k<8$' but not $\sqrt{64}$.	
	2		

$kx^2+2x-k=kx-2$ leading to $kx^2+(-k+2)x-k+2$ [$=0$]	*M1	Eliminate y and form 3-term quadratic. Allow 1 error.	Nov 2021 /11/Q2
$(-k+2)^2-4k(-k+2)$	DM1	Apply b^2-4ac; allow 1 error but a, b and c must be correct for *their* quadratic.	
$5k^2-12k+4$ or $(-k+2)(-k+2-4k)$	A1	May be shown in quadratic formula.	
$(-k+2)(-5k+2)$	DM1	Solving a 3-term quadratic in k (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give *their* coefficient of k^2.	
$\frac{2}{5}<k<2$	A1	WWW, accept two separate correct inequalities. If M0 for solving quadratic, **SC B1** can be awarded for correct final answer.	
	5		

Intersection of a line and a quadratic curve

Substitute for y (or x) in first equation and simplify	*M1	All terms to one side and brackets expanded.	Nov 2024 /11/Q4
Obtain $10x^2+3kx-40$ [$=0$] (or $10y^2+11ky+k^2-360$[$=0$])	A1		
Attempt b^2-4ac for 3-term quadratic involving k	DM1	Not in quadratic formula unless b^2-4ac is isolated.	
Obtain $9k^2+1600$ (or $81k^2+14400$)	A1		
$9k^2+1600>0$	A1 FT	FT for $ak^2+b>0$ with $a, b>0$.	
	5		

March 2024 /12/Q7

(a)	Attempt substitution for y in quadratic equation	*M1	Or substitution for x …
	Obtain $5x^2+30x+75-k[=0]$ or $5y^2-20y+50-k[=0]$	A1	OE e.g. $x^2+6x+15-\frac{k}{5}$ (all terms gathered together).
	Use $b^2-4ac=0$ with *their* a, b and c	DM1	'= 0' may be implied in subsequent working or the answer.
	Obtain $900-20(75-k)=0$ or equivalent and hence $k=30$	A1	… obtaining $400-20(50-k)=0$ and $k=30$.
		4	
(b)	Substitute *their* value of k in equation from part **(a)** and attempt solution	M1	Expect $5x^2+30x+45[=0]$ or $5y^2-20y+20[=0]$.
	Obtain coordinates $(-3,\ 2)$	A1	**SC B1** only $(-3,\ 2)$ without attempt at quadratic solution.
		2	

March 2023 /12/Q1

$x^2-kx+2=3x-2k$ leading to $x^2-x(k+3)+(2+2k)[=0]$	M1	3-term quadratic, may be implied in the discriminant.
$b^2-4ac=(k+3)^2-8(1+k)$ (ignore '= 0' at this stage)	DM1	Cannot just be seen in the quadratic formula.
$=(k-1)^2$ accept $(k-1)(k-1)$	A1	Or use of calculus to show minimum of zero at $k=1$ or sketch of $f(k)=k^2-2k+1$.
$\geqslant 0$ Hence will meet for all values of k	A1	Clear conclusion.
	4	

March 2022 /12/Q2

$x^2+2cx+4=4x+c$ leading to $x^2+2cx-4x+4-c\ [=0]$	*M1	Equate ys and move terms to one side of equation.
$b^2-4ac=(2c-4)^2-4(4-c)$	DM1	Use of discriminant with *their* correct coefficients.
$[4c^2-16c+16-16+4c=]\ 4c^2-12c$	A1	
$b^2-4ac>0$ leading to $(4)c(c-3)>0$	M1	Correctly apply '> 0' considering both regions.
$c<0,\ c>3$	A1	Must be in terms of c. **SC B1** instead of M1A1 for $c \leqslant 0,\ c \geqslant 3$
	5	

Nov 2021 /11/Q2

June 2021 /13/Q1

(a)	$(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$	B1	$k(4x-3)^2$ where $k \neq 1$ scores B0 but mark final answer, allow recovery.
	$+1$ or $b=1$	B1	
		2	
(b)	[For one root] $k=1$ or '*their b*'	B1 FT	Either by inspection or solving or from $24^2-4\times16\times(10-k)=0$ WWW
	[Root or $x=$]$\frac{3}{4}$ or 0.75	B1	**SC B2** for correct final answer WWW.
		2	

$x^2+kx+6=3x+k$ leading to $x^2+x(k-3)+(6-k)\,[=0]$	**M1**	Eliminate y and form 3-term quadratic.	March 2021 /12/Q4
$(k-3)^2-4(6-k)[>0]$	**M1**	OE. Apply b^2-4ac.	
$k^2-2k-15[>0]$	**A1**	Form 3-term quadratic.	
$(k+3)(k-5)[>0]$	**A1**	Or $k=-3,\ 5$ from use of formula or completing square.	
$k<-3,\quad k>5$	**A1 FT**	Or any correct alternative notation, do not allow $\leqslant, \geqslant$. FT for *their* outside regions.	
	5		

1	$4(x-3)^2$ seen or $a=4$ and $b=-3$	**B1**	OE Award marks for the correct expression or their values a, b and c. Condone $4(x-3)+p-36=0$ and $4\left(\frac{p}{4}-9\right)$.
	$-36+p$ or $p-36$ seen or $c=p-36$	**B1**	
		2	
	$p-36>0$ leading to $p>36$ or $24^2-4\times4p\{0\Rightarrow p\}36$ or $36<p$	**B1**	Allow $(36,\infty)$ or $36<p<\infty$. Consider final answer only.
		1	

9709/ June2023/12/Q3

2	$[8x^6+215x^3-27=0]$ leading to $(8x^3-1)(x^3+27)[=0]$ **OR** $\frac{-215\pm\sqrt{215^2-4.8.-27}}{2.8}$ or $\frac{-215\pm\sqrt{47089}}{2.8}$	**M1**	OE If a substitution is used then the correct coefficients must be retained. Condone substitution of $x=x^3$.
	$\frac{1}{8}$, -27	**A1**	Both correct values seen. **SC:** if M0 scored **SC B1** is available for sight of $\frac{1}{8}$ and -27 OE
	$\frac{1}{2}$ or $0.5, -3$	**A1**	**SC:** if M0SCB1 scored then **SCB1** is available for the correct answers and no others. Do not ISW if answers given as a range.
		3	

9709/ June2023/12/Q4

3	$x^2-kx+2=3x-2k$ leading to $x^2-x(k+3)+(2+2k)\,[=0]$	**M1**	3-term quadratic, may be implied in the discriminant.
	$b^2-4ac=(k+3)^2-8(1+k)$ (ignore '= 0' at this stage)	**DM1**	Cannot just be seen in the quadratic formula.
	$=(k-1)^2$ accept $(k-1)(k-1)$	**A1**	Or use of calculus to show minimum of zero at $k=1$ or sketch of $f(k)=k^2-2k+1$.
	$\geqslant 0$ Hence will meet for all values of k	**A1**	Clear conclusion.
		4	

9709/ March2023/12/Q1

4	$k^2 - 4\times8\times2$ [< 0]	M1	Use of $b^2 - 4ac$ but not just in the quadratic formula.
	$-8 < k < 8$ or $-8 < k$, $k < 8$ or $\lvert k\rvert < 8$ or (–8, 8)	A1	Condone '$-8 < k$ or $k < 8$', '$-8 < k$ and $k < 8$' but not $\sqrt{64}$.
		2	

9709/ Nov2022/12/Q3

5	(a)	$x^2 - 8x + 11 = (x-4)^2 \ldots$ or $p = -4$	B1	If p and q-values given after *their* completed square expression, mark the expression and ISW.
		... –5 or $q = -5$	B1	
			2	
	(b)	$(x-4)^2 - 5 = 1$ so $(x-4)^2 = 6$ so $x - 4 = [\pm]\sqrt{6}$	M1	Using *their* p and q values or by quadratic formula
		$x = 4 \pm \sqrt{6}$ or $\frac{8 \pm \sqrt{24}}{2}$	A1	Or **exact** equivalent. No FT; must have ± for this mark. ISW decimals 1.55, 6.45 if exact answers seen. If M0, **SC B1** possible for correct answers.
			2	

9709/ June2022/11/Q1

6	(a)	$\{(x+1)^2 + 2(x+1) - 5\} + \{3\}$, or $\{(x+1+1)^2\} + \{-6+3\}$	M1 M1	M1 for dealing with $\begin{pmatrix}-1\\0\end{pmatrix}$ and M1 for dealing with $\begin{pmatrix}0\\3\end{pmatrix}$.
		$[y =] x^2 + 4x + 1$	A1	Answer only given full marks.
			3	
	(b)	{Stretch} {x direction or horizontally or y-axis invariant} { factor ½}	B2, 1, 0	Additional transformation B0.
			2	

9709/ June2022/13/Q4

7	(a)	$6y + 2 - 7y^{1/2}$ [= 0]	*M1	OE Rearrange to a 3-term quadratic.
		$\left(2y^{\frac{1}{2}} - 1\right)\left(3y^{\frac{1}{2}} - 2\right)$ [= 0] or e.g. $(2u-1)(3u-2)$ [= 0]	DM1	Or use of formula or completing the square.
		$[y^{1/2} =] \frac{1}{2}, \frac{2}{3}$	A1	Answers only **SC B1** if DM1 not scored.
		$[y =] \frac{1}{4}, \frac{4}{9}$	A1	Answers only **SC B1** if DM1 not scored.
			4	
	(b)	Use of $\tan x$ = *their* y values	M1	Must have at least 2 values of y from part **(a).**
		x = 14[.0], 24[.0], x = 194[.0], 204[.0]	A1 A1 FT	FT for 180 + angle (twice). AWRT
			3	

9709/ June2022/13/Q5

8	$x^2 + 2cx + 4 = 4x + c$ leading to $x^2 + 2cx - 4x + 4 - c$ [= 0]	*M1	Equate ys and move terms to one side of equation.
	$b^2 - 4ac = (2c-4)^2 - 4(4-c)$	DM1	Use of discriminant with *their* correct coefficients.
	$[4c^2 - 16c + 16 - 16 + 4c =] 4c^2 - 12c$	A1	
	$b^2 - 4ac > 0$ leading to $(4)c(c-3) > 0$	M1	Correctly apply '> 0' considering both regions.
	$c < 0$, $c > 3$	A1	Must be in terms of c. **SC B1** instead of M1A1 for $c \leqslant 0$, $c \geqslant 3$
		5	

9709/ March2022/12/Q2

9	(a)	$2[\{(x-2)^2\}\ \{+3\}]$	**B1 B1**	B1 for $a=2$, B1 for $b=3$. $2(x-2)^2+6$ gains B1B0
			2	

9709/ March2022/12/Q5

10	$kx^2+2x-k=kx-2$ leading to $kx^2+(-k+2)x-k+2\ [=0]$	***M1**	Eliminate y and form 3-term quadratic. Allow 1 error.
	$(-k+2)^2-4k(-k+2)$	**DM1**	Apply b^2-4ac ; allow 1 error but a, b and c must be correct for *their* quadratic.
	$5k^2-12k+4$ or $(-k+2)(-k+2-4k)$	**A1**	May be shown in quadratic formula.
	$(-k+2)(-5k+2)$	**DM1**	Solving a 3-term quadratic in k (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give *their* coefficient of k^2.
	$\frac{2}{5}<k<2$	**A1**	WWW, accept two separate correct inequalities. If M0 for solving quadratic, **SC B1** can be awarded for correct final answer.
		5	

9709/ Nov2021/11/Q2

11	(a)	$\{-3(x-2)^2\}\quad\{+14\}$	**B1 B1**	B1 for each correct term; condone $a=2$, $b=14$.
			2	

9709/ Nov2021/11/Q8

12	(a)	$\{5(y-3)^2\}\quad\{+5\}$	**B1 B1**	Accept $a=-3$, $b=5$
			2	

9709/ Nov2021/13/Q3

13	(a)	$(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$	B1	$k(4x-3)^2$ where $k\neq 1$ scores B0 but mark final answer, allow recovery.
		$+1$ or $b=1$	B1	
			2	
	(b)	[For one root] $k=1$ or '*their b*'	B1 FT	Either by inspection or solving or from $24^2-4\times16\times(10-k)=0$ WWW
		[Root or $x=$]$\frac{3}{4}$ or 0.75	B1	**SC B2** for correct final answer WWW.
			2	

9709/ June2021/12/Q1

14	$x^2-4x+3=mx-6$ leading to $x^2-x(4+m)+9$	***M1**	Equating and gathering terms. May be implied on the next line.
	b^2-4ac leading to $(4+m)^2-4\times9$	**DM1**	SOI. Use of the discriminant with *their* a, b and c
	$4+m=\pm6$ or $(m-2)(m+10)=0$ leading to $m=2$ or -10	**A1**	Must come from $b^2-4ac=0$ SOI
	Substitute both *their m* values into *their* equation in line 1	**DM1**	
	$m=2$ leading to $x=3$; $m=-10$ leading to $x=-3$	**A1**	
	(3, 0), (–3, 24)	**A1**	Accept 'when $x=3$, $y=0$; when $x=-3$, $y=24$' If final A0A0 scored, **SC B1** for one point correct WWW

9709/ June2021/13/Q1

15	$x^2+kx+6=3x+k$ leading to $x^2+x(k-3)+(6-k)\ [=0]$	**M1**	Eliminate y and form 3-term quadratic.
	$(k-3)^2-4(6-k)[>0]$	**M1**	OE. Apply b^2-4ac.
	$k^2-2k-15[>0]$	**A1**	Form 3-term quadratic.
	$(k+3)(k-5)[>0]$	**A1**	Or $k=-3,\ 5$ from use of formula or completing square.
	$k<-3,\quad k>5$	**A1 FT**	Or any correct alternative notation, do not allow $\leqslant, \geqslant$. FT for *their* outside regions.
		5	
			9709/ March2021/12/Q4