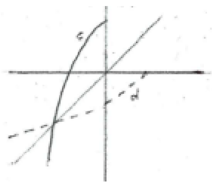
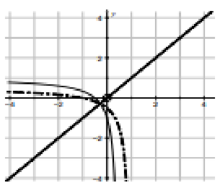
Revision – Unit2 Functions

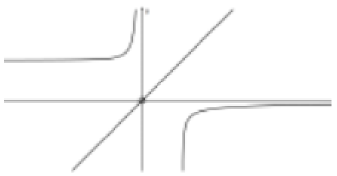
(5 types of questions)

1-Type of Functions

(a)	Obtain $b=2$ and $c=\frac{3}{2}$	**B1**		Nov 2024 /11/Q11
	Obtain $\frac{15}{2}-2\left(x-\frac{3}{2}\right)^2$	**B1**		
	State range is $y \leqslant \frac{15}{2}$ or $f(x) \leqslant \frac{15}{2}$ with $\leqslant$ given or clearly implied (not <)	**B1 FT**	Following *their* value of *a*.	
		3		
(b)	State that reflection is in *x*-axis	**B1**	Accept transformations in any order.	
	State or imply that translation is by $\begin{pmatrix} -\frac{3}{2} \\ \frac{15}{2} \end{pmatrix}$ or equivalent	**B1 FT**	Following *their* values of *a* and *c* in part **(a)**. Accept transformations in any order.	
		2		
(c)	Sketch the correct graph appearing in second and third quadrants only	**B1**		
	State that each *y*-value is associated with a single *x*-value or equivalent	**B1**	Accept passes the horizontal line test. Ignore passes the vertical line test.	
		2		
(d)	Sketch the correct graph with suitable labelling to distinguish the two curves	**B1**	Appearing in third and fourth quadrants only.	
	Draw the line $y=x$	**B1**	See above; no need to label the line.	
	Attempt correct process for finding the inverse function	**M1**	Allowing use of $\pm$ and *y* so far.	
	Obtain $\frac{3}{2}-\sqrt{\frac{15}{4}-\frac{1}{2}x}$ or equivalent	**A1**	Must involve *x* at the conclusion.	
		4		

(a)(i)	$[\mathrm{f}(-1)=]\ \frac{1}{3}$	**B1**	Condone 0.333.	Nov 2024 /12/ Q5
		1		
(a)(ii)		**B1**	For showing the correct mirror line.	
		B1	For correct shape: the curves should intersect in the first square in the third quadrant. To the left of the point of intersection, the reflection is below the original and crosses the *x*-axis. To the right of the point of intersection, the reflection is to the right the original.	
		2		

(a)(iii)	$\frac{2x+1}{2x-1}=y \Rightarrow 2x+1=y(2x-1)$	M1*	Equating y to the given function and clearing of fractions. x and y may be interchanged at this stage.	
	$2xy-2x=y+1$	DM1	Condone ± errors during simplification.	
	$\frac{x+1}{2(x-1)}, \frac{-x-1}{2-2x}$	A1	Allow 'f^{-1}' or '$y =$' but NOT '$x =$', nor fractions within fractions.	
	[Domain of f^{-1} is] $x<1$	B1	Accept $-\infty<x<1$ or $(-\infty, 1)$, condone $[-\infty, 1)$.	
	Alternative Method for Question 5(a)(iii)			
	$y=1+\frac{2}{2x-1} \Rightarrow y-1=\frac{2}{2x-1}$	M1*	Equating y to the given function after division by $2x-1$. Isolating the term in x. x and y may be interchanged at this stage.	
	$2x=\frac{2}{y-1}+1$	DM1	Condone ± errors during simplification.	
	$\frac{1}{x-1}+\frac{1}{2}$	A1	OE Allow 'f^{-1}' or '$y =$' but NOT '$x =$', nor fractions within fractions.	
	[Domain of f^{-1} is] $x<1$	B1	Accept $-\infty<x<1$ or $(-\infty, 1)$, condone $[-\infty, 1)$.	
		4		
b)	$gf\left(\frac{1}{4}\right)=-7$	B1		
	$\frac{2x+1}{2x-1}=-7$	M1	Equating $\frac{2x+1}{2x-1}$ to *their* $gf\left(\frac{1}{4}\right)$.	
	$[x=]\ \frac{3}{8}$	A1	OE	
	Alternative solution for Question 5(b)			
	$gf\left(\frac{1}{4}\right)=-7$	B1		
	$x=f^{-1}(-7)$	M1	$x=f^{-1}\left(\textit{their } gf\left(\frac{1}{4}\right)\right)$	
	$[x=]\ \frac{3}{8}$	A1	OE	
		3		

Question	Answer	Marks	Guidance	Source
(a)	$3(x-2)^2+2$ or $a=-2,\ b=2$	B1 B1		Nov 2024 /13/Q8
		2		
(b)	2 or $k=2$ or $k\geq 2$	B1FT	FT on *their a*. Do not accept $x=2$ or $x \geqslant 2$.	
		1		
(c)	$3(x-2)^2+14-12=y \Rightarrow (x-2)^2=\frac{y-2}{3}$	M1	Using *their* completed square form.	
	$x=[\pm]\sqrt{\frac{y-2}{3}}+2$	DM1		
	$f^{-1}(x)=\sqrt{\frac{x-2}{3}}+2$	A1	OE, e.g. $y=\frac{\sqrt{3x-6}}{3}+2$.	
		3		
d)	Finding $f^{-1}(29)$ [= 5]	M1	Or solving f(*x*) = 29 [using *their* completed square form, OE].	
	Finding $f^{-1}($*their* 5$)$	M1	Or solving f(*x*) = *their* 5.	
	$x=3$	A1	If using f(*x*) method, $x=1$ must be discarded.	
	Alternative solution for Question 8(d)			
	$3\left(3(x-2)^2+2\right)-2)^2+2=29$ using *their* completed square form	M1	Or $3\left(3x^2-12x+14\right)^2-12\left(3x^2-12x+14\right)+14=29$. Allow if the '$=29$' appears later in the working.	
	Solving as far as $9(x-2)^4=9$ or $x^2-4x+3=0$	DM1	OE Or $[27]\left(x^4-8x^3+24x^2-32x+15\right)=0$.	
	$x=3$ only	A1	WWW Only dependent on the first M1.	
		3		
(b)	Obtaining $\left(3(5x-1)-2\right)^2+$ *their k*	M1	May simplify and/or use *k* at this stage; *k* may have come from an inequality in **(a)**.	Mar2024 /12/Q9
	Conclude $[\text{fg}(x)]\geqslant 8$ allow $[y]\geqslant 8$	A1 FT	OE Following *their* value of *k*; must be $\geqslant$, not $>$. Allow an accurate written description.	
		2		
(c)	State $g^{-1}(x)=\frac{1}{5}(x+1)$	B1	OE $\frac{1}{5}(x+1)$ must be indicated as the inverse.	
	$[h(x)=]7x+4$	B1B1	If $7x+4$ only, it must be clear that this is $h(x)$.	
		3		
(a)		B1	For curve in correct quadrant.	May2024 /11/Q6
		B1	Fully correct including line $y=x$. Horizontal asymptote closer to *x* axis than vertical asymptote is to *y* axis.	
		2		

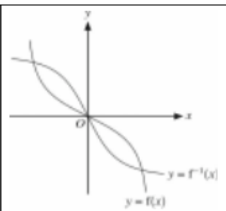
Question	Answer	Marks	Guidance	
(b)	$x=\frac{2}{y^2}+4$ leading to $y^2(x-4)=2$ or $y^2=\frac{2}{(x-4)}$	M1	Allow x and y swapped around.	
	$y^2=\frac{2}{(x-4)}$ leading to $y=[\pm]\sqrt{\frac{2}{x-4}}$ or $x=[\pm]\sqrt{\frac{2}{y-4}}$	M1		
	$\left[f^{-1}(x)\right]=-\sqrt{\frac{2}{x-4}}$	A1		
		3		
(c)	$[x]=-2$	B1		
		1		
(d)	Because f^{-1} is always negative and f is always positive or curves do not intersect	B1	Accept other correct answers e.g. 'f is only defined for positive values of x and f^{-1} is only defined for negative values of x' or 'domains do not overlap' or 'the y values cannot be the same' or 'the x values cannot be the same'.	
		1		
(a)	$\left[f^{-1}(x)=\right](x+1)^2$	B1	ISW Condone '$y=$'.	May2024/12/Q4
		1		
(b)	$0<g(x)\leqslant\frac{1}{2}$ or $g(x)>0$ and $g(x)\leqslant\frac{1}{2}$ or $\left(0,\frac{1}{2}\right]$	B1	Do not allow $g(x)>0, g(x)\leqslant\frac{1}{2}$. Do not allow $g(x)>0$ or $g(x)\leqslant\frac{1}{2}$. Condone g or y in place of $g(x)$.	
	g^{-1} does not exist because it is one to many or g^{-1} does not exist because it is not one to one. Or g^{-1} does not exist because g is not one to one or g^{-1} does not exist because g is many to one or g^{-1} does not exist because g fails the horizontal line test.	B1	g^{-1} can be replaced by 'It' throughout. A correct statement followed by any further incorrect explanation can be awarded B1.	
		2		
(c)	$f\left(\frac{25}{16}\right)=\frac{1}{4}$	B1	SOI	
	$\frac{1}{\left(\sqrt{x}-1\right)^2+2}=\frac{1}{4}$	M1	Equating $\frac{1}{\left(\sqrt{x}-1\right)^2+2}$, or *their* 'simplified' version, to *their* $f\left(\frac{25}{16}\right)$.	
	$\left[\left(\sqrt{x}-1\right)^2+2=4 \text{ leading to}\right]$ $\sqrt{x}-1=\sqrt{2}$ leading to $x=\left(1\pm\sqrt{2}\right)^2$ Or $\left[x-2\sqrt{x}+1+2=4 \text{ leading to}\right]$ $x-2\sqrt{x}-1=0$ leading to $x=\left(1\pm\sqrt{2}\right)^2$ Or $\left[x-1=2\sqrt{x} \text{ leading to}\right]$ $x^2-6x+1=0$ leading to $x=\frac{6\pm\sqrt{36-4}}{2}$	A1	Simplification as far as $x=\ldots$ Allow just + in the results because − can be disregarded at this stage. Can be implied by the final answer. Note: $x=1\pm\sqrt{2}$ scores A0.	
	$3+2\sqrt{2}$	A1	Must discount the solution $3-2\sqrt{2}$.	
		4		

(a)	Express $f(x)$ as: $a-(x-3)^2$ or $a-(3-x)^2$ where $a=\pm19$ or ±1	M1	OE If the form $-f(x)=(x^2-6x-10)$ is used the form must be returned to $f(x)=\ldots$ Completed square form must give $-x^2$. Answers must come from completion of the square (not calculus or graphs).	May2024/13/Q11
	$19-(3-x)^2$ or $19-(x-3)^2$	A1	OE	
	$f(x)\leqslant19$ or $y\leqslant19$ with $\leqslant$, not $<$ or $-\infty<f(x)\leqslant19$ or $-\infty\leqslant f(x)\leqslant19$ or $(-\infty, 19]$ or $[-\infty, 19]$	A1 FT	Using *their* constant following the award of M1. **SC B1** answer only or answer from a method not involving completion of the square.	
		3		
(b)	$g^{-1}(x)=\frac{1}{4}(x-k)$	B1		
	$g^{-1}f(x)=\frac{1}{4}(10+6x-x^2-k)=4x+k$	M1	OE May use *their* completed square form for f(x).	
	Simplify the quadratic equation obtained from $g^{-1}f(x)=g(x)$ provided k is present and apply $b^2-4ac=0$ to this quadratic equation	*M1	Expect $x^2+10x-10+5k=0$.	
	Obtain $100-4(5k-10)=0$ and hence $k=7$	A1		
	Use *their* k to form and solve a quadratic in x	DM1	Allow if *their* quadratic has two solutions.	
	$(-5,-13)$ only	A1	**SC B1** if no method seen.	
	Alternative Method for first 4 marks			
	State $f(x)=gg(x)$	(B1)		
	$gg(x)=16x+5k$	(M1)		
	Apply $b^2-4ac=0$ to quadratic equation obtained from $f(x)=gg(x)$	(*M1)	Provided k is present.	
	$100-4(5k-10)=0$ and hence $k=7$	(A1)		
		6		
	Stretch: $(2x)^2-2(2x)+5$ or $(x-1)^2+4$ leading to $(2x-1)^2+4$	M1	Replacing x by $2x$.	Mar2024/13/Q2
	Reflection: $(-2x)^2-2(-2x)+5$ or $(-2x-1)^2+4$	M1	Replacing x by $-x$. FT on *their* stretch.	
	Stretch: $3\{(-2x)^2-2(-2x)+5\}$ or $3\{(-2x-1)^2+4\}$	M1	Multiplying the whole function by 3. FT on *their* (stretch plus reflection).	
	$12x^2+12x+15$	A1		
		4		

2- The Domain and Range of a function

Question	Answer	Marks	Guidance	Source
(a)	$[y]\leqslant -1$	B1	Accept f or $f(x)\leqslant -1$, $-\infty<y\leqslant -1$, $(-\infty,-1]$. Do not accept $x\leqslant -1$.	Mar2023 /12/ Q9
		1		
(b)	$y=-3x^2+2$ rearranged to $3x^2=2-y$, leading to $x^2=\frac{2-y}{3}$ or $y^2=\frac{2-x}{3}$	M1		
	$x=[\pm]\sqrt{\frac{2-y}{3}} \rightarrow [f^{-1}(x)]=\{-\}\left\{\sqrt{\frac{2-x}{3}}\right\}$	A1 A1	A1 for minus, A1 for $\sqrt{\frac{2-x}{3}}$, allow $-\sqrt{\frac{x-2}{-3}}$.	
		3		
(c)	$fg(x)=-3(-x^2-1)^2+2$	M1	SOI expect $-3x^4-6x^2-1$.	
	$gf(x)=-(-3x^2+2)^2-1$	M1	SOI expect $-9x^4+12x^2-5$.	
	$fg(x)-gf(x)+8=0$ leading to $6x^4-18x^2+12\ [=0]$	A1	OE	
	$[6](x^2-1)(x^2-2)\ [=0]$ or formula or completion of the square	M1	Solving a 3-term quadratic equation in x^2 must be seen.	
	$x=-1\ ,-\sqrt{2}$ only these **two** solutions	A1	Allow $-\sqrt{1}$, $-1.41[4]$ Answers only **SC B1**.	
		5		
	Stretch: $(2x)^2-2(2x)+5$ or $(x-1)^2+4$ leading to $(2x-1)^2+4$	M1	Replacing x by $2x$.	Mar 2023 /12/ Q2
	Reflection: $(-2x)^2-2(-2x)+5$ or $(-2x-1)^2+4$	M1	Replacing x by $-x$. FT on *their* stretch.	
	Stretch: $3\{(-2x)^2-2(-2x)+5\}$ or $3\{(-2x-1)^2+4\}$	M1	Multiplying the whole function by 3. FT on *their* (stretch plus reflection).	
	$12x^2+12x+15$	A1		
		4		
5(a)	Three points at the bottom of their transformed graph plotted at $y=2$	B1	All 5 points of the graph must be connected.	Nov2022 /12/ Q5
	Bottom three points of /\/\ at $x=0$, $x=1$ & $x=2$	B1	Must be this shape.	
	All correct	B1	Condone extra cycles outside $0\leqslant x\leqslant 2$.	
		3	**SC:** If B0 B0 scored, B1 available for /\ in one of correct positions **or** all 5 points correctly plotted and not connected **or** correctly sized shape in the wrong position.	
5(b)	$[g(x)=]\ f(2x)+1$	B1 B1	Award marks for their final answer as follows: f(2x) B1, + 1 B1. Condone $y=$ or $f(x)=$.	
(a)	$[f(x)]=\{-2(x+2)^2\}-\{5\}$	B1 B1		Nov2022 /13/ Q2
		2		
(b)	$[f(x)]<-7$	B1	Allow $y<-7$, <-7, $(-\infty,-7)$ or less than -7 $-\infty\langle f(x)\langle -7, -7\rangle f(x)\rangle -\infty$, $f<-7$	
		1		
(c)	$y=-2(x+2)^2-5 \rightarrow (x+2)^2=\frac{-(y+5)}{2}$	M1	Operations correct. Allow sign errors. FT *their* quadratic from **(a)**.	
	$x=[\pm]\sqrt{\frac{-(y+5)}{2}}-2$	M1	Operations correct. Allow sign errors. FT *their* quadratic from **(a)**.	
	$[f^{-1}(x)]=-2-\sqrt{\frac{-(x+5)}{2}}$ or $-2-\sqrt{-\frac{(x+5)}{2}}$	A1	Allow $[f^{-1}(x)]=-2-\sqrt{\frac{x+5}{-2}}$.	
		3		

)(a)	$x \neq 1$ or $x < 1,\ x > 1$ or $(-\infty,1),(1,\infty)$ $[x \in \mathbb{R}]$	B1	Must be x not $f^{-1}(x)$ or y. Do not accept $1 < x < 1$.	June2022 /12/ Q10
		1		
)(b)	$y = \frac{2x+1}{2x-1}$ leading to $(2x-1)y = 2x+1$ leading to $2xy - y = 2x+1$	*M1	Setting $y =$, removing fraction and expanding brackets.	
	$2xy - 2x = y+1$ leading to $2x(y-1) = y+1$ leading to $x = \frac{y+1}{2(y-1)}$	DM1	Reorganising to get $x =$. Condone ± sign errors only.	
	$[f^{-1}(x)] = \frac{x+1}{2(x-1)},\ \frac{x+1}{x-1} \times \frac{1}{2}$ or $\frac{1}{x-1} + \frac{1}{2}$	A1	OE. Must be in terms of x. Do not allow $\frac{x+1}{x-1} \div 2$.	
		3		
)(c)	$(\textit{their } f^{-1}(3))$ leading to $(\textit{their } f^{-1}(3))^2 + 4$ $[f^{-1}(3) = 1, 1+4 =]$	M1	Correct order of operations and substitution of $x = 3$ needed.	
	5	A1		
		2		
)(d)	Sight of 'not one to one' or 'many to one' or 'one to many'	B1	Any reason mentioning 2 values, or + and —, such as: square root gives 2 values or horizontal line test crosses curve twice or 2 values because of turning point or 2 values because it is a quadratic.	
		1		
0(e)	$f(x) = 1 + \frac{2}{2x-1} = \frac{2x-1}{2x-1} + \frac{2}{2x-1} = \frac{2x+1}{2x-1}$	B1	AG Do not condone equating expressions and verification.	
	$f'(x) = -4(2x-1)^{-2}$ or $2(2x-1)^{-1} + \{-(2x+1)2(2x-1)^{-2}\}$ or $\frac{(2x-1)2 - 2(2x+1)}{(2x-1)^2}$	*M1	For $k(2x-1)^{-2}$ and no other terms or correct use of the product or quotient rule then ISW.	
	Gradient $m = -4$	A1	Differentiation must have clearly taken place.	
	Equation of tangent is $y - 3 = -4(x-1)$ $[\Rightarrow y = -4x + 7]$	DM1	Using (1, 3) in the equation of a line with *their* gradient.	
	Crosses axes at $\left(\frac{7}{4}, 0\right)$ and $(0, 7)$	A1 FT	SOI from *their* straight line or by integration from 0 to '*their* 7/4'.	
	[Area =] $\frac{49}{8}$	A1	OE e.g. 6.13 AWRT. If M0 A0 DM0, **SC B2** available for correct answer.	
		6		
(b)	{Translation} $\begin{pmatrix}\{2\}\\ \{3\}\end{pmatrix}$ OR {Stretch} {y direction} {factor 2}	B2,1,0	B2 for fully correct, B1 with two elements correct. {} indicates different elements.	Mar2022 /12/ Q5
	{Stretch} {y direction} {factor 2} OR {Translation} $\begin{pmatrix}\{2\}\\ \{6\}\end{pmatrix}$	B2,1,0	B2 for fully correct, B1 with two elements correct. {} indicates different elements.	
		4		
(b)	$[k =]\ 2$	B1	Allow $[x] \leqslant 2$.	Nov2021 /11/ Q8
		1		

(c)	[Range is] $[y] \leqslant -13$	B1	Allow $[f(x)] \leqslant -13$, $[f] \leqslant -13$ but NOT $x \leqslant -13$.	
		1		
(d)	$y=-3(x-2)^2+14$ leading to $(x-2)^2=\frac{14-y}{3}$	M1	Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.	
	$x=2(\pm)\sqrt{\frac{14-y}{3}}$	A1	Allow $\frac{y-14}{-3}$.	
	$[f^{-1}(x)]=2-\sqrt{\frac{14-x}{3}}$	A1	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root <u>only.</u>	
	Alternative method for question 8(d)			
	$x=-3(y-2)^2+14$ leading to $(y-2)^2=\frac{14-x}{3}$	M1	Allow $\frac{x-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.	
	$=2(\pm)\sqrt{\frac{14-x}{3}}$	A1	Allow $\frac{x-14}{-3}$.	
	$[f^{-1}(x)]=2-\sqrt{\frac{14-x}{3}}$	A1	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root <u>only.</u>	
		3		
(e)	$[g(x)=]\ \{-3(x+3-2)^2\}+\{14+1\}$	B2, 1, 0	OR $\{-3(x+3)^2\}+\{12(x+3)\}+\{3\}$	
	$g(x)=-3x^2-6x+12$	B1		
		3		
(a)	$f(5)=[2]$ and $f(\textit{their}\ 2)=[5]$ OR $ff(5)=\left[\frac{2+3}{2-1}\right]$ OR $\dfrac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1}$ and an attempt to substitute $x=5$.	M1	Clear evidence of applying f twice with $x=5$.	Nov2021 /12/ Q3
	5	A1		
		2		
(b)	$\frac{x+3}{x-1}=y \Rightarrow x+3=xy-y$ OR $\frac{y+3}{y-1}=x \Rightarrow y+3=xy-x$	*M1	Setting $f(x)=y$ or swapping x and y, clearing of fractions and expanding brackets. Allow ± sign errors.	
	$xy-x=y+3 \Rightarrow x=\frac{y+3}{y-1}$ OE OR $y+3=xy-x \Rightarrow y=\left[\frac{x+3}{x-1}\right]$ OE	DM1	Finding x or $y=$. Allow ± sign errors.	
	$[f^{-1}(x) \text{ or } y]=\frac{x+3}{x-1}$	A1	OE e.g. $1+\frac{4}{x-1}$ etc. Must be a function of x, cannot be $x=$.	
		3		
(a)		B1	A reflection of the given curve in $y=x$ (the line $y=x$ can be implied by position of curve).	Nov2021 /13/ Q6
		1		

(b)	$y=\frac{-x}{\sqrt{4-x^2}}$ leading to $x^2=y^2\left(4-x^2\right)$	*M1	Squaring and clearing the fraction. Condone one error in squaring $-x$ or y	
	$x^2\left(1+y^2\right)=4y^2$	DM1	OE. Factorisation of the new subject with order of operations correct. Condone sign errors.	
	$x=(\pm)\frac{2y}{\sqrt{1+y^2}}$	DM1	$x=(\pm)\sqrt{\left(\frac{4y^2}{(1+y^2)}\right)}$ OE is acceptable for this mark. Isolating the new subject. Order of operations correct. Condone sign errors.	
	$f^{-1}(x)=\frac{-2x}{\sqrt{1+x^2}}$	A1	Selecting the correct square root. Must not have fractions in numerator or denominator.	
		4		
(c)	1 or $a=1$	B1	Do not allow $x=1$ or $-1<x<1$	
		1		
(d)	$[\text{fg}(x)=\text{f}(2x)=]\ \frac{-2x}{\sqrt{4-4x^2}}$	B1	Allow $\frac{-2x}{\sqrt{4-(2x)^2}}$ or any correct unsimplified form.	
	$\text{fg}(x)=\frac{-x}{\sqrt{1-x^2}}$ or $\frac{-x}{1-x^2}\sqrt{1-x^2}$ or $\frac{x}{x^2-1}\sqrt{1-x^2}$	B1	Result of cancelling 2 in numerator and denominator.	
		2		
				Nov2021 /13/ Q6

3- Inverse Functions

4-Composite Functions

5- Transformations of functions
